Journal of Physics:
Conference Series

PURPOSE-LED PUBLISHING™

PAPER • **OPEN ACCESS**

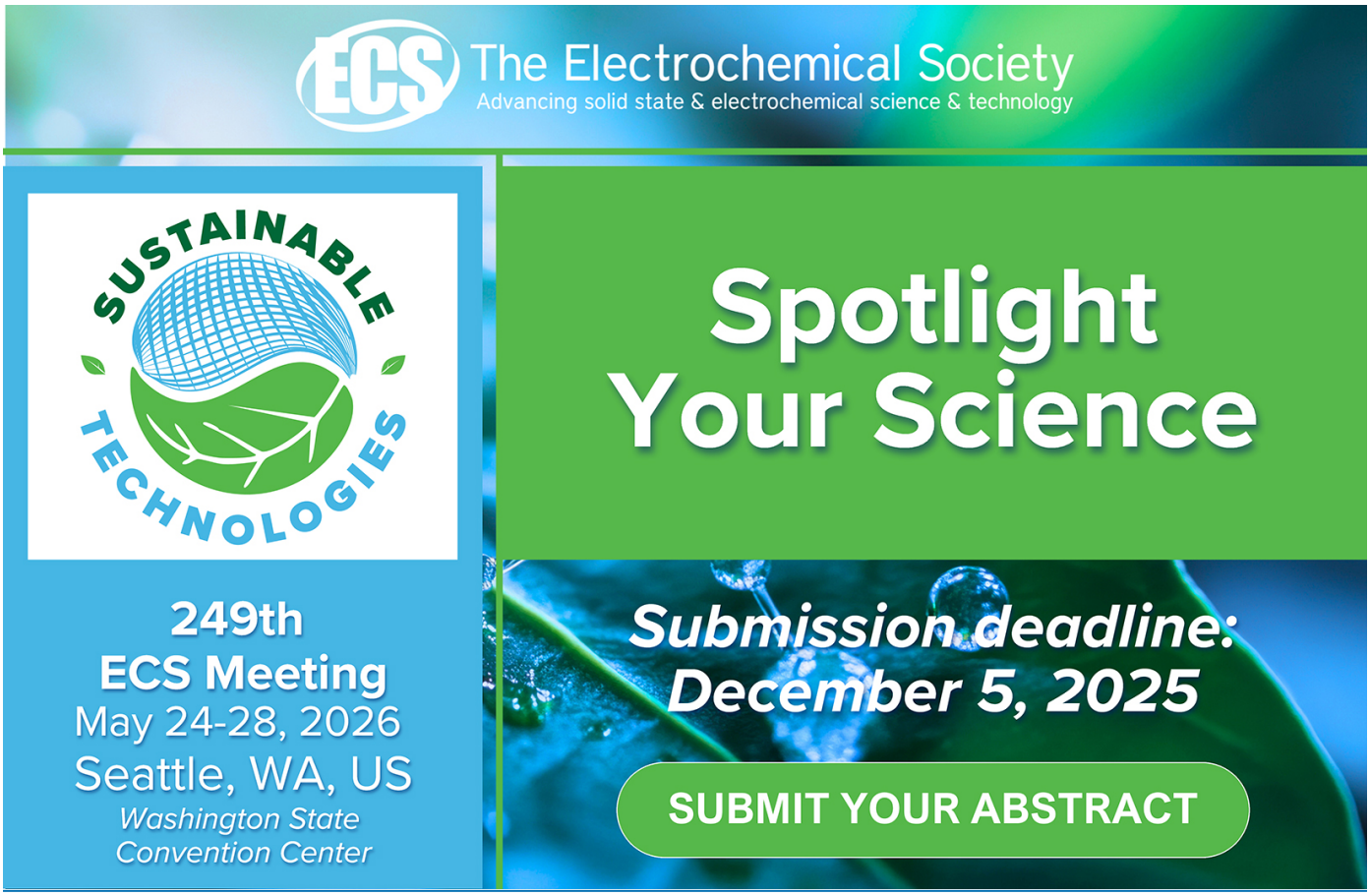
Reinforced concrete beams subjected to three-point bending using finite element method

To cite this article: N Afanador García *et al* 2021 *J. Phys.: Conf. Ser.* **1938** 012008

View the article online for updates and enhancements.

You may also like

- Quantification of uncertainty in steel plates subject to fatigue with variable load via chaos expansion polynomial
N Afanador García, C J Noriega Sanchez and G Guerrero G
- Evaluation of uncertainty in determining the physical properties of concrete using Bootstrap
N Afanador García, C J Noriega Sanchez and C Nolasco Serna
- TEM Analyses of Unusual Presolar Silicon Carbide: Insights into the Range of Circumstellar Dust Condensation Conditions
S. A. Singerling, N. Liu, L. R. Nittler et al.



This content was downloaded from IP address 8.243.64.131 on 28/10/2025 at 15:56

Journal of Physics: Conference Series **1938** (2021) 012008 doi:10.1088/1742-6596/1938/1/012008

Reinforced concrete beams subjected to three-point bending using finite element method

N Afanador García[1], G Guerrero Gómez[2], and C Nolasco Serna[3]

[1] Grupo de Investigación en Ingeniería Civil, Universidad Francisco de Paula Santander, Seccional Ocaña, Colombia

[2] Grupo de Investigación en Tecnologia y Desarrollo en Ingeniería, Universidad Francisco de Paula Santander, Seccional Ocaña, Colombia

[3] Grupo de Investigación Facultad de Educación Arte y Humanidades, Universidad Francisco de Paula Santander, Seccional Ocaña, Colombia

E-mail: nafanadorg@ufpso.edu.co, gguerrerog@ufpso.edu.co

Abstract. Buildings are generally composed of reinforced concrete elements and understanding the response of these elements to the actions to which they are subjected is crucial for the design of a feasible, safe, and economic structure. This paper analyzes 3D finite element modeling using the commercial software ANSYS for a reinforced concrete beam subjected to three-point bending, simply supported with a doubly reinforced cross-section and transverse reinforcement. This work validated the results of a numerical model with theoretical results for a reinforced concrete beam subjected to monotonic loading. This work was carried out for the civil engineering program of Universidad Francisco de Paula Santander, Ocaña, Colombia, in order to reduce student attrition, increase motivation, and broaden the professional skills of future graduates.

1. Introduction

Reinforced concrete is one of the most important construction materials, widely used in many types of engineering structures, such as: buildings, bridges, hydroelectric and nuclear power plants, treatment plants, containment structures, wharfs, among others. The economy, efficiency, strength and stiffness of reinforced concrete make it an attractive material for a wide range of structural applications, in addition to generating direct and indirect employment. Understanding the behavior of structural elements during applied action is fundamental for the design of a reliable, feasible, and safe structure. There are different methodologies to study the response of structural elements. Experimentally, the behavior of reinforced concrete beams is studied, which is performed on a large scale. Experimental results are compared with theoretical calculations that estimate the internal distribution of tensile and compressive stresses, as well as deflections and deformations in beams. Another type of analysis can be performed using finite element modeling to estimate the behavior of the element and to compare the experimental results with those obtained using the finite element method, which provides a complement to the experimental results.

In the design of beams subjected to bending, the tensile reinforcement must be designed to meet the ductility requirements, so that, if collapse occurs, it is due to tension in the reinforcing steel, that is, a ductile failure. In this way, brittle failure resulting from crushing of the concrete


Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd

in the compression zone should be avoided; from the experimental analysis on concrete beams with simple reinforcement, it is concluded that the ductility decreases as the tensile reinforcement increases [1]. Buckhouse [2] compared the theoretical results obtained for the flexural behavior of reinforced concrete beams with those found experimentally. Wolanski [3] simulated and validated the experimental results provided by Buckhouse [2] using commercial finite element software ANSYS. Fanning [4] used reinforced concrete beams of 3.0 m and cross-section of 155 mm x 240 mm to validate the results found in the finite element analysis using ANSYS, finding that these are sensitive to the Young's modulus of concrete [5-7] and to the elastic limit of the concrete reinforcement. Dahmani, *et al.* [8] used Solid65 element with a smeared reinforcement approach to study crack propagation in reinforced concrete beams using ANSYS software.

In this paper, the model uses a discrete and linear material approach all developed in ANSYS V21 R1 finite element software [9]. The model was used to evaluate the maximum compressive stress in concrete, as well as the deflection at the center of the span. The validity of the model is calibrated with the results of the elastic analysis using the transformed-section method.

2. Finite element model of reinforced concrete beam

The analysis by means of the finite element method allows us to provide solutions to problems that would be laborious to obtain theoretically and would be costly to carry out experimentally. The numerical modeling was performed with the commercial software [9]. This software has powerful numerical simulation programs based on the finite element method, which can solve structural engineering problems ranging from linear to nonlinear analysis, among other problems. A reinforced concrete beam with a point load at the center of the span was analyzed, see Figure 1(a), and the longitudinal and transverse reinforcement is shown in Figure 1(b).

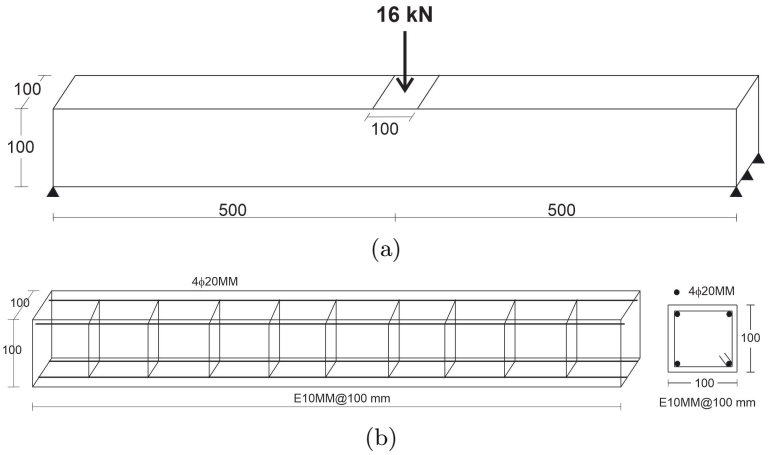


Figure 1. Model of reinforced concrete beam, (a) loading and boundary conditions and (b) schematic representation of the longitudinal and transverse reinforcement of the beam.

The analysis of a beam subjected to point load at the center using ANSYS [9] is performed in three stages, as follows.

i. Preprocessing, development of the finite element model and the environmental factors applied to it, such as: boundary conditions, forces, symmetries, etc.
ii. Solution of the analysis, solution of the finite element model.
iii. Post-processing, calculation of deflections and stresses.

There are three techniques for modeling steel reinforcement in the finite element model for reinforced concrete: the discrete model (Figure 2(a)), the embedded model (Figure 2(b)), and the smeared model (Figure 2(c)). In the discrete model, the reinforcement uses bar or beam elements that are connected to the nodes of the concrete mesh, see Figure 2(a).

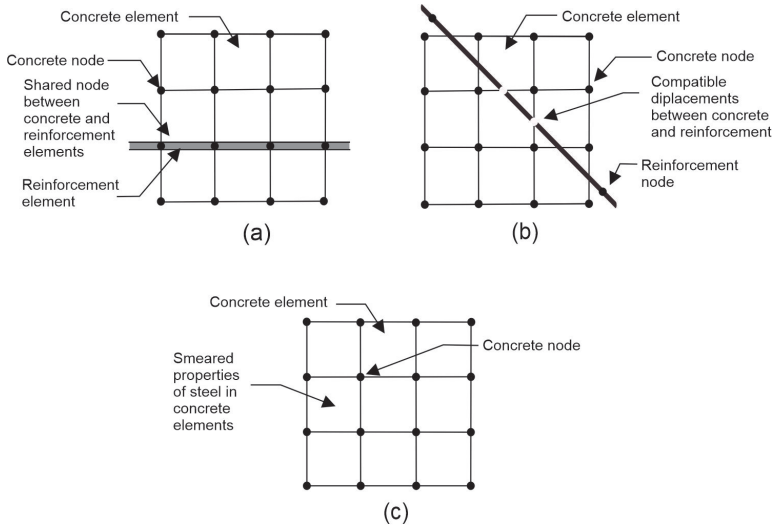


Figure 2. Models for reinforcement of reinforced concrete materials: (a) discrete model, (b) embedded model, and (c) smeared model. Adapted from [10].

In this problem, a discrete model was used to model the beam reinforcement. where the concrete and the reinforcement mesh share the same nodes and the concrete occupies the same regions as the reinforcement. A disadvantage of this model is that the concrete mesh is constrained by the location of the reinforcement. The reinforced concrete beam material was modeled with solid elements type Solid185 with eight nodes with three degrees of freedom for each node; translational degrees of freedom in the node directions x, y, and z. This element has the ability to deform plastically, cracking in the nodal directions up to concrete crushing [11]; a schematic of the Solid185 element is shown in Figure 3(a).

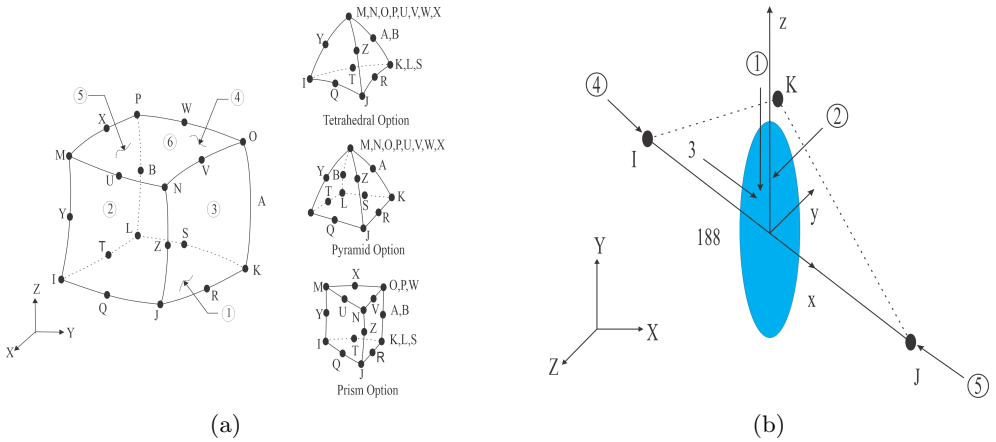
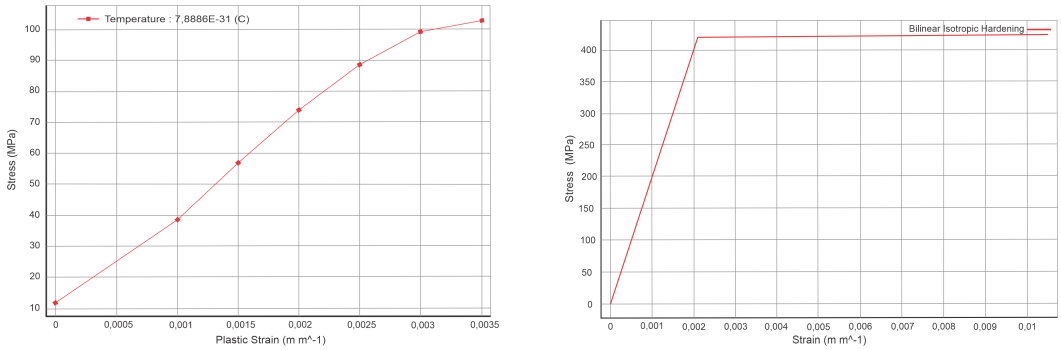


Figure 3. Model of reinforced concrete beam, (a) loading and boundary conditions and (b) schematic representation of the longitudinal and transverse reinforcement of the beam.

The Solid185 element is suitable for modeling 3D solid structures; the use of prismatic degeneracies such as tetrahedrons, pyramids and prisms can be used for irregular regions, see Figure 3(a). A Beam188 element was used to model the reinforcing steel, see Figure 3(b). This element is a 3D long element, which has two nodes with seven degrees of freedom, three translational and three rotational in the x, y, and z nodal direction for each node, plus one degree of freedom for deformation (optional). This element is suitable for linear applications of large rotations, and large nonlinear deformations. This element is based on the Timoshenko beam theory that includes shear deformation effects [12], which supports models of elasticity, plasticity, creep, and other nonlinear materials. The model is composed of two materials: concrete defined by a multilinear hardening model, see Figure 4(a), with modulus of elasticity, $E_c = 35217.2$ N/mm^2, uniaxial compressive strength, $f'_c = 41$ N/mm^2 and Poisson's ratio $= 0.25$. The steel exhibits bilinear behavior, shown in Figure 4(b), with elastic modulus, $E_s = 2E + 05$ N/mm^2, yield stress, $f_y = 420$ N/mm^2 and Poisson$'$s ratio $= 0.30$.



(a) (b)

Figure 4. Behavior of concrete and steel, (a) multilinear hardening model of concrete, and (b) bilinear model of reinforcing steel.

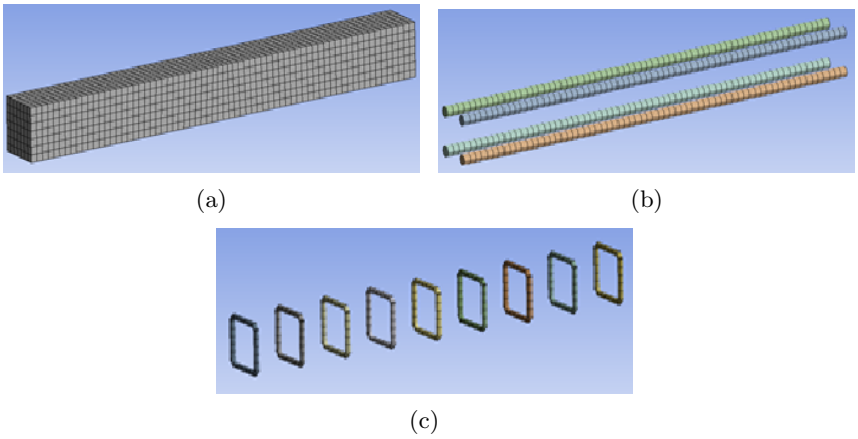
In the finite element model, boundary conditions of fixed support (left support) and displacement (right support) were applied, necessary to achieve a unique solution. The force of 160 kN is distributed on the top of the beam in a cross-section of 100 mm x 100 mm. The maximum size of the finite elements for the concrete beam was 15 mm, 10 mm for the longitudinal reinforcement (Ref.) and finally for the transverse reinforcement was 5 mm; statistics of nodes and elements are given in Table 1.

Table 1. Statistics of nodes and elements of the numerical model.

Description	Concrete beam	Longitudinal Ref.	Transverse Ref.
Nodes	16496	17036	17468
Elements	3332	3600	3816

The Figure 5 shows the discretization of the model, which reflects the statistics of the model presented in Table 1. Figure 5(a) shows the discretization of the concrete beam, Figure 5(b) shows the discretization of the longitudinal reinforcement, and Figure 5(c) shows the discretization of the transverse reinforcement.

As a result of numerical modeling using ANSYS and for theoretical verification purposes, the normal stress in the z-direction is presented, see Figure 6.



(a) (b)

(c)

Figure 5. Discretization of the model, (a) concrete beam, (b) longitudinal reinforcement and (c) transverse reinforcement.

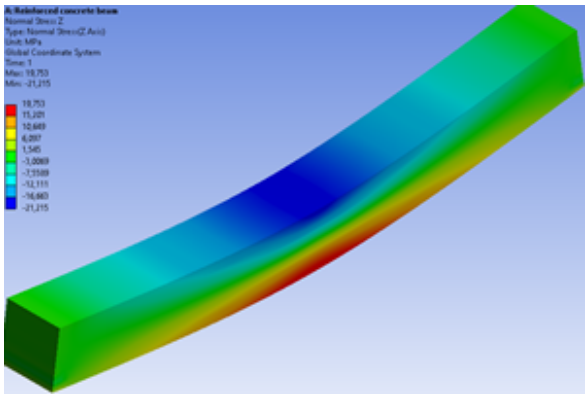


Figure 6. Normal stresses in the z-direction.

The normal stress corresponds to the expected results for a simply supported beam subjected to three-point bending, with maximum compressive and tensile stress at the mid-span section. The displacement estimated according to the numerical model is shown in Figure 7, with a maximum displacement of 1.61 mm at the mid-span section.

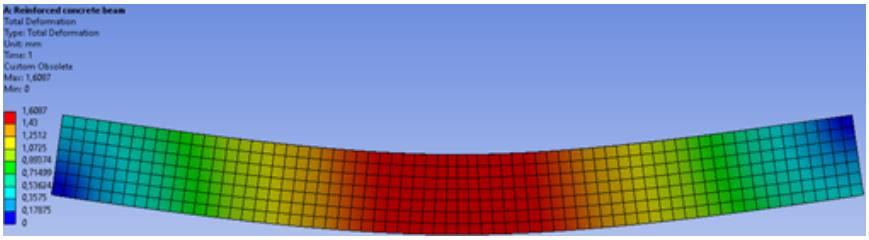


Figure 7. Displacement in the Z-direction.

The maximum value of the displacement estimated according to the model is validated with respect to the theoretical value, evaluated in the following section.

3. Results and discussions

In the following subsections, the maximum tensile stress in concrete present in the middle of the free span between supports will be analyzed, using ANSYS [9] and manual calculation.

3.1. Stress along Z-direction

Maximum tensile stress in concrete is 19.80 N/mm^2, given in the finite element model using ANSYS [9], see Figure 6.

3.2. Manual calculation

The tensile and compressive reinforcement is replaced by a theoretical concrete that resists as much as the reinforcement to achieve a transformed or homogeneous section, see Figure 8.

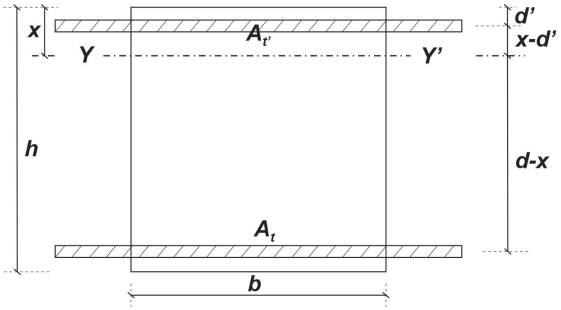


Figure 8. Transformed cross-section of a doubly reinforced rectangular beam.

The calculation of the maximum compressive stress in concrete is evaluated theoretically to compare with the result of the numerical model made in ANSYS [9]. The maximum bending moment (M) for the present boundary conditions, is defined in Equation (1).

$$M = \frac{PL}{4}, \tag{1}$$

where, P corresponds to the applied point load (N) and L represents the distance between supports (mm). The modulus of elasticity of concrete (E_c) can be expressed as a function of the compressive strength of the concrete (f'_c), see Equation (2).

$$E_c = 5500(f'_c)^{\frac{1}{2}}. \tag{2}$$

The modulus of elasticity of reinforcing steel (E_s) is defined as $2E+05$ N/mm^2, therefore the modular relation (n) is established as $n = E_s/E_c$. Applying the transformed section method, the equivalent increased area of concrete at tension zone (A_t), is declared in the Equation (3).

$$A_t = nA_s, \tag{3}$$

where, A_s is the tensile steel area. The Equivalent increased area of concrete at compression zone (A'_t) is defined as $(2n-1)A'_s$, then taking moments about the neutral axis, it is possible to establish the Neutral axis depth (x) by solving the Equation (4).

$$bx^2/2 + A'_t(x - d') = A_t(d - x), \tag{4}$$

where, b is the width of the cross section of the beam, d' represents the distance between the centroid of the compression reinforcement and the concrete surface furthest in compression, and finally d is the effective height of the cross section. The Transformed moment of inertia ($I_{Y-Y'}$) is defined in the Equation (5).

$$I_{Y-Y'} = (bx^3)/3 + (2n-1)2(\pi\phi_{sup}^4)/64 + A_t'(x-d')^2 + n(\pi\phi_{inf}^4)/64 + A_t(d-x)^2, \quad (5)$$

where, ϕ_{sup} and ϕ_{inf} represents the diameter of the upper and lower reinforcing steel respectively. The Maximum compressive stress in concrete is established using the bending equation, presented in the Equation (6).

$$f_c = (Mx)/I_{Y-Y}. \quad (6)$$

The numerical values of Equation (1) to Equation (6) are presented in Table 2.

Table 2. Transformed section results.

Description	Results
M (N·mm)	4.00E+06
E_c (N/mm^2)	35217.20
n	5.68
A_t (mm^2)	1465.44
A_t' (mm^2)	2672.88
x (mm)	29.37
$I_{Y-Y'}$ (mm^4)	5972.48
f_c (N/mm^2)	19.67

The relative difference between the maximum tensile stresses estimated by ANSYS [9] and the theoretical one is 0.41%. The deflection at the centerline of the concrete beam is 1.61 using the finite element method, while using the theory of elasticity expressed in Equation (7).

$$\Delta = \frac{PL^3}{48E_cI_{Y-Y}} = 1.58\ mm \quad (7)$$

The relative difference between the estimated maximum displacement using the numerical model and the theoretical one is 1.86%.

4. Conclusions

Analysis by means of finite element models in reinforced concrete beams, using discrete models in ANSYS, allowed to obtain the response to elastic bending with precision. The estimated results of stress and deflection in the center of the free span of the beam are very close to those found by a theoretical analysis. This analysis served to motivate and reduce the desertion of the students of the solid mechanics subject of the civil engineering program and propose more applications in topics such as: deflection considering shear force and temperature changes.

References

[1] Hasan M W, El-Khatieb M, Al-Ani H 2011 Steel reinforcement ratio dependency of plastic rotational capacity of reinforced concrete beams *Jordan Journal of Civil Engineering* **5(4)** 480-491
[2] Buckhouse E 1997 *External Flexural Reinforcement of Existing Reinforced Concrete Beams Using Bolted Steel Channels* (United States of America: Marquette University) pp 1-95
[3] Wolanski A J 2004 *Flexural Behavior of Reinforced and Prestressed Concrete Beams Using Finite Element Analysis* (United States of America: Marquette University) pp 1-87
[4] Fanning P 2001 Nonlinear models of reinforced and post-tensioned concrete beams *Electronic Journal of Structural Engineering* **2(1)** 111-119
[5] Evangelista F, Afanador-García N 2016 A polynomial chaos expansion approach to the analysis of uncertainty in viscoelastic structural elements *DYNA* **83(199)** 172-182

[6] Afanador N, Noriega Sanchez C J, Nolasco Serna C 2020 Evaluation of uncertainty in determining the physical properties of concrete using Bootstrap *Journal of Physics: Conference Series* **1645(1)** 012008:1-7
[7] Afanador N, Sanjuan Y, Medina D 2012 Diseño sísmico de muros de contención en gravedad y voladizo *Revista Colombiana de Tecnologias de Avanzada* **2(20)** 97-104
[8] Dahmani L, Khennane, Kaci S 2010 Crack identification in reinforced concrete beams using ANSYS software *Strength of Materials* **42(2)** 232-340
[9] ANSYS Inc 2021 *User's Manual ANSYS V21 R1* (United States of America: ANSYS Inc.)
[10] Srinivasan R, Sathiya K 2010 Flexural behavior of reinforced concrete Beams using finite element analysis (elastic analysis) *Buletinul Institutului Politehnic Din Lasi. Sectia Constructii, Arhitectura* **56(4)** 1-31
[11] Adheem A H 2013 Nonlinear analysis of reinforced concrete beams strengthened in shear with NSM FRP rods *Journal of Babylon University / Engineering Sciences* **21(1)** 160-173
[12] Timoshenko S, Delgado T 1967 *Resitencia de Materiales: Teoria Elemental y Problemas* (España: Espasa-Calpe)