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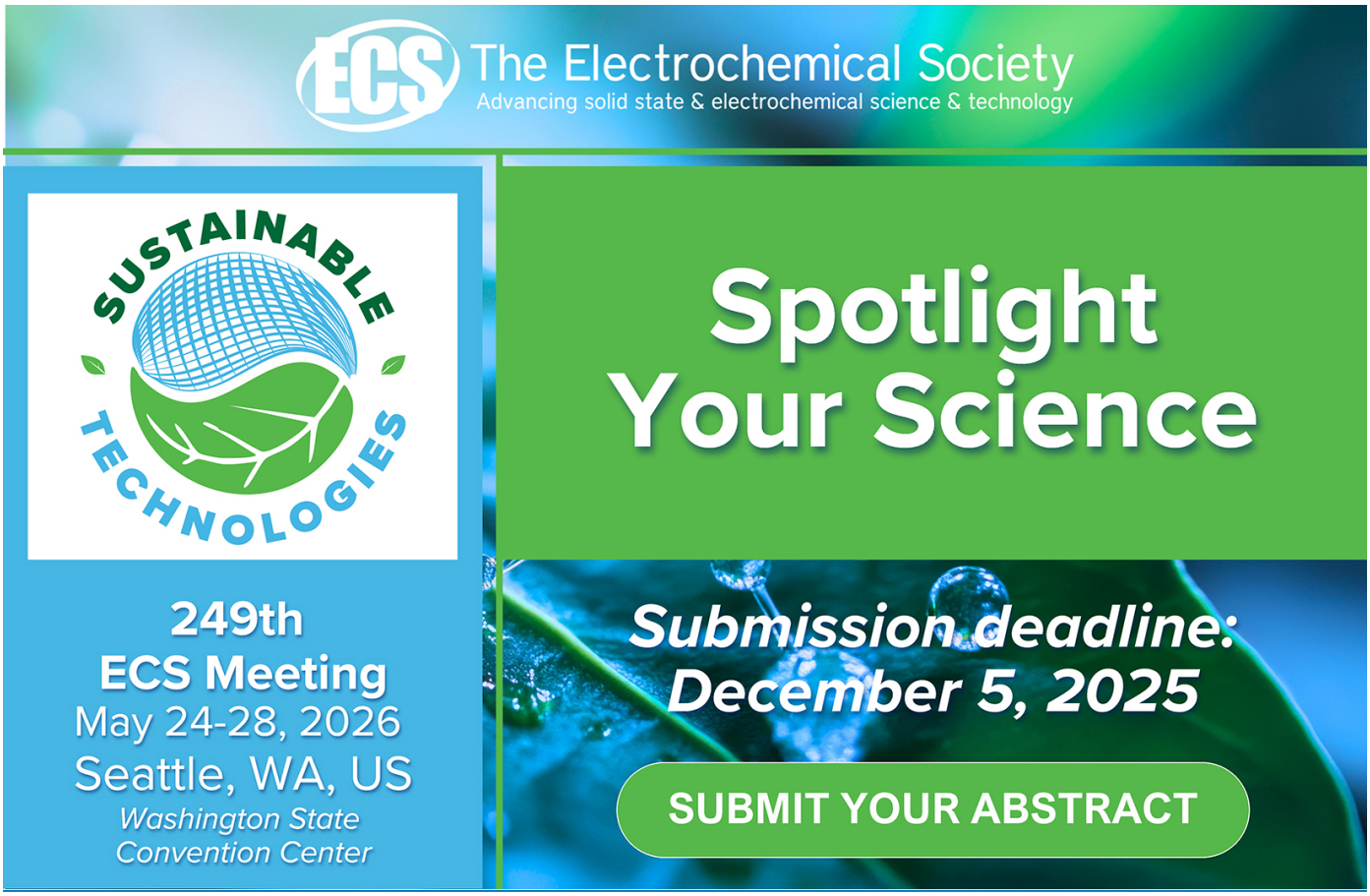
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Applications of the method of lines for modeling the physical phenomenon of heat conduction

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Abstract. It is claimed that most of the fundamental principles that govern the physical phenomena of interest in engineering applications can be described by differential equations. Therefore, the ability to analyze, solve and understand differential equations is essential for decision-making in the applied areas. In this sense, studying efficient methods to solve differential equations is a fundamental contribution in advancing the understanding of relevant physical models in engineering applications. The purpose of this research allowed to study the differential equation associated with Fourier's heat transfer law and calculate its solution by two different methods than the traditional method. From this context, the study is related to the phenomenon of heat conduction in a metal bar under ideal conditions to later carry out its application in a particular case. First, the solution of the differential equation that models the physical phenomenon derived from the use of Fourier's physical heat law is calculated with the use of statistical tools; then, a solution scheme is implemented using method of lines. Given the nature of the investigation, the solutions by both methods are compared about what is expected in the physical interpretation of Fourier's law.

1. Introduction

Mathematics is playing an increasingly important role in physics and the life sciences, showing a resurgence of interest in modern and classical applied mathematical techniques [1]. The application of mathematical models in understanding the physical phenomenon of heat conduction has a major impact on applications throughout the world [2]. An important conceptual place to start the study of mathematical models related to heat transfer is directly related to Fourier theory [3]. The study of heat conduction phenomena in kilns has had a wide interest in the research [4], the mechanical engineering department of Universidad Francisco de Paula Santander, Ocaña, Colombia, has devoted research to the study of heat transfer processes in industrial and artisanal kils [5].

The search for numerical techniques to solve mathematical models associated with heat conduction processes is important because of their computational implementation. In contrast, analytical solutions have elaborate developments that are difficult to process algorithmically [6]. The main contribution of this article is the use of the method of lines as a numerical tool to solve the heat equation that models the heat conduction phenomenon. In addition, it is verified that the method of lines coincides with a

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small tolerance with the traditional method of solution of the thermal conduction model [7]. It is important to mention that this article is directly related to the study of thermal phenomena in the region of influence, Ocaña, Colombia [8,9].

2. Mathematical modeling of heat conduction

The process of mathematical modeling describes several stages. The definition of the physical phenomenon and the empirical relationships it describes, this generates a mathematical model which must be solved by different mathematical techniques. Finally, the interpretation and verification of the data.

2.1. Formulation of the physical phenomena

A metal bar 100 cm long, whose sides maintain constant temperatures of 0 ℃ and 100 ℃, is put in contact with another metal bar that has a temperature of 0 ℃. The problem is to calculate the temperature across the bar after the cooling process starts.

Since this problem describes a phenomenon of heat conduction in one dimension [10], the appropriate mathematical model for this situation is the partial derivative Equation (1).

$$\frac{\partial \mathrm{T}}{\partial \mathrm{t}} = \alpha^2 \frac{\partial^2 \mathrm{T}}{\partial \mathrm{x}^2}. \tag{1}$$

The function $\mathrm{T}(\mathrm{x},\mathrm{t})$ represents the temperature in the bar, where x is the position and t the time. The constant α represents the thermal diffusivity of the bar. The initial and boundary conditions are defined in complete Equation (2) and Equation (3).

$$\mathrm{T}(0,0) = 0,\ \mathrm{T}(100,0) = 100^{\circ}, \tag{2}$$

$$\mathrm{T}(\mathrm{x},0) = \mathrm{f}(\mathrm{x}) = \mathrm{x},\ \ 0 < \mathrm{x} < 100. \tag{3}$$

Equation (1) and Equation (3) represent the mathematical model to calculate the temperature in the bar.

2.2. Analytical solution for the heat conduction equation

Initially it is assumed that the temperature function $\mathrm{T}(\mathrm{x},\mathrm{t}) = \mathrm{g}(\mathrm{x})\mathrm{h}(\mathrm{t})$, this method is called separation of variables [11]. When calculating the partial derivatives with respect to the variables x and t, in the previous equality, it is possible to conclude complete Equation (4) and Equation (5).

$$\frac{\partial \mathrm{T}}{\partial \mathrm{t}} = \mathrm{g}(\mathrm{x})\mathrm{h}'(\mathrm{t}), \tag{4}$$

$$\frac{\partial^2 \mathrm{T}}{\partial \mathrm{x}^2} = \mathrm{g}''(\mathrm{x})\mathrm{h}(\mathrm{t}). \tag{5}$$

Substituting Equations (4) and Equation (5) in Equation (1) we have Equation (6).

$$\frac{\mathrm{h}'}{\alpha^2 \mathrm{h}} = \frac{\mathrm{g}''}{\mathrm{g}} = \beta^2, \tag{6}$$

where the constant β^2 is the result of equating two functions that depend on different variables. The Equation (6) allows the definition of two ordinary differential equations whose solutions [11] are expressed in complete Equation (7) and Equation (8).

$$\mathrm{h}(\mathrm{t}) = \mathrm{A}e^{-\alpha^2\beta^2\mathrm{t}}, \tag{7}$$

$$g(x) = B\cos(\beta x) + C\text{sen}(\beta x). \tag{8}$$

When applying the Fourier integral technique [12] it is natural to assume the Equation (9).

$$T(x,t) = \int_{-\infty}^{\infty}[T_1(t)\cos(\beta x) + T_2(t)\,\text{sen}(\beta x)]d\beta. \tag{9}$$

Using the complete Equation (9) by the Fourier integral method [7], it is possible to compute the general solution Equation (10).

$$T(x,t) = \frac{1}{\sqrt{\pi}}[x\int_0^{y_2} e^{-y^2}dy - x\int_0^{y_1} e^{-y^2}dy - \alpha\sqrt{t}(e^{-{y_2}^2} - e^{-{y_1}^2})]. \tag{10}$$

2.3. Method of lines

The numerical method chosen to solve the heat conduction model Equation (1) and Equation (3) is method of lines [13]. The first step to use the lines method consists in discretizing the equation Equation (1), for this aim it is possible to apply the Taylor's theorem Equation (11).

$$T(x+h) = f(x) + f'(x)h + \cdots + \frac{1}{(n+1)!}f^{(n+1)}(x)h^{n+1}. \tag{11}$$

Applying the complete Equation (11) to the derivative $\frac{\partial^2 T}{\partial x^2}$ yields the approximation Equation (12).

$$T(x+h) = f(x) + f'(x)h + \cdots + \frac{1}{(n+1)!}f^{(n+1)}(x)h^{n+1}. \tag{12}$$

Substituting the complete Equation (12) in Equation (1) produces a set of recursive complete Equation (13).

$$T_j'(t) = \frac{1}{h^2}(T_{j-1}(t) - 2T_j(t) + T_{j+1}(t). \tag{13}$$

Using Equation (13), the boundary conditions Equation (2) and the initial condition Equation (3), the mathematical model for heat conduction Equation (1) and Equation (3) can be represented in the matrix form Equation (14).

$$T'(t) = AT(t) + g(t), \tag{14}$$

where, the Equation (15) is.

$$A = \frac{1}{h^2}\begin{pmatrix} -2 & 1 & 0 & 0 & \cdots & 0 \\ 1 & -2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 1 & -2 & 1 \\ 0 & 0 & \cdots & 0 & 1 & -2 \end{pmatrix}. \tag{15}$$

Is a tridiagonal matrix and Equation (16).

$$g(t) = \frac{1}{h^2}\begin{pmatrix} g_0(t) \\ \vdots \\ g_1(t) \end{pmatrix}. \tag{16}$$

3. Results and discussion

In this section, the temperature along the 100 cm long metal bar will be calculated after 900 seconds have already elapsed. We assume that $\alpha^2 = 0.173$, the estimate of the temperature along the bar is calculated to $x = 0.25$ cm, 50 cm, 75 cm and 100 cm.

3.1. Analytical method

From [7], it is known that $y_1 = 0, -1, -2, -3, -4$; furthermore, $y_2 = 4, 3, 2, 1, 0$. By using the estimation of the normal distribution [14] it is possible to compute the integrals in the complete Equation (17) and Equation (18).

$$\frac{2}{\sqrt{\pi}}\int_0^1 e^{-y^2}dy = 0.84270, \int_0^2 e^{-y^2}dy = 0.99532, \tag{17}$$

$$\frac{2}{\sqrt{\pi}}\int_0^3 e^{-y^2}dy = 0.99998, \frac{2}{\sqrt{\pi}}\int_0^4 e^{-y^2}dy = 0.99999. \tag{18}$$

From [7], by substituting the values of the integrals in the complete Equation (17) and Equation (18) it is possible in the temperature function of complete Equation (10) to calculate the following values in the complete Equation (19) and Equation (20).

$$T(0,900) = 7.05°C,\ T(25,900) = 25.6°C, \tag{19}$$

$$T(50,900) = 49.77°C,\ T(75,900) = 66.5°C,\ T(100,900) = 42.9°C. \tag{20}$$

3.2. Lines method

By applying complete Equations (14), Equation (15) and Equation (16) to the study of the change in temperature of the metal bar with the initial and boundary conditions of Equation (2) and Equation (3), for the case n = 3, has the matrix form, show in the Equation (21). The numerical method for solving the system of ordinary is the Rungen Kutta method [15].

$$\frac{\partial T}{\partial t} = \begin{pmatrix} \widehat{T_1} \\ \widehat{T_2} \\ \widehat{T_3} \end{pmatrix} = \frac{\alpha}{\Delta x^2}\begin{pmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix}\begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} + \frac{\alpha}{\Delta x^2}\begin{pmatrix} 0 \\ 0 \\ 100 \end{pmatrix} \tag{21}$$

3.3. Analytical method and method of lines

Applying the Lagrange interpolation method to the temperature values the complete Equation (19) and Equation (20) generates the temperature curve (red) that is illustrated in Figure 1. When solving the Equation (21) using a computer program, a set of curves are generated that represent the temperature of the bar as time passes. The curve in Figure 1 (blue) represents the temperature when the time is 900 seconds.

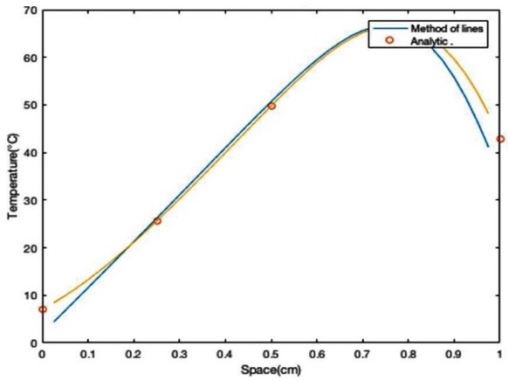


Figure 1. Contour of temperature with the method of lines and analytical method.

From [7], it is possible to conclude that the analytical method fits the expected result for the physical behavior of the temperature variation in the bar. Therefore, from the qualitative behavior shown by Figure 1 it is possible to conclude that the method of line is a suitable candidate for modeling heat conduction on homogeneous media.

4. Conclusion

The study of the cooling of the metal bar can be calculated in two different ways. First, a solution of the heat equation is proposed by using the techniques of Fourier analysis to calculate the temperature function. Second, the solution by method of lines of the conduction phenomena is proposed by the numerical scheme. The advantage of the solution by the numerical method of lines is the possibility of the calculation by means of a computer program. As a general conclusion to the work, for the modeling of thermal processes, where the diffusion constant is homogeneous, method of lines is an efficient alternative for this type of phenomena.

References

[1] Lin C C, Segel L A 1988 *Mathematics Applied to Deterministic Applied to Deterministic Problems in the Natural Sciences* (Philadelphia: Siam)
[2] Markowich P A 2007 *Applied Partial Differential* **(**New York: Springer Verlag)
[3] Carslow H S, Jaeger I C 1979 *Conduction of Heat in Solids*, 2nd ed (New York: Oxford University)
[4] Milani M, Montorsi L, Stefani M, Saponelli R, Lizzano M 2017 Numerical analysis of an entire ceramic kiln under actual operating conditions for the energy efficiency improvement *Jornal of Environmental Management* **203** 1026
[5] Guerrero Gómez G, Espinel Blanco E, Sánchez Acevedo H G 2017 Análisis de temperaturas durante la cocción de ladrillos macizos y sus propiedades finales *Revista Tecnura* **21(51)** 118
[6] Skrzypczak E W, Skrzypczak T 2017 Analytical and numerical solution of the heat conduction problem in the rod *Journal of applied Math and Comp Mechanics* **16(4)** 79
[7] Rice D 1952 A problem in heat conduction and its solution *Am Journal Phys* **20** 263
[8] Nolasco C, Guerrero Gómez G, Gómez J A 2019 Mathematical model of firing process of ladrillera Ocaña, Colombia *Journal of. Physics: Conference Series* **1408** 012017:1
[9] Nolasco C, Jacome N J, Hurtado N A 2019 Solution by numerical methods of the heat equation in engineering applications. Mathematical A case of study: Cooling without the use of electricity *Jornal of Physics: Conference Series* **1388** 012017:1
[10] Logan D 2015 *Applied Partial Differential Equations* (New York: Springer Verlag)
[11] Baehr H D, Stephan K 2015 *Heat and Mass Transfer* (New York: Springer Verlag)
[12] Kreyszig E 2006 *Advanced Engineering Mathematics* (Singapore: John Wiley & Sons)
[13] LeVeque R 2007 *Finite Difference Methods for Ordinary and Partial Differential Equations* (Philadelphia: Siam)
[14] Mood A, Graybill F 1978 *Introducción a la Teoria de la Estadistica* (Madrid: Aguilar)
[15] Burden R, Faires J 2010 *Análisis Numérico* (México: Editorial Cengage Learning)