Journal of Physics:
Conference Series

PURPOSE-LED PUBLISHING™

PAPER • **OPEN ACCESS**

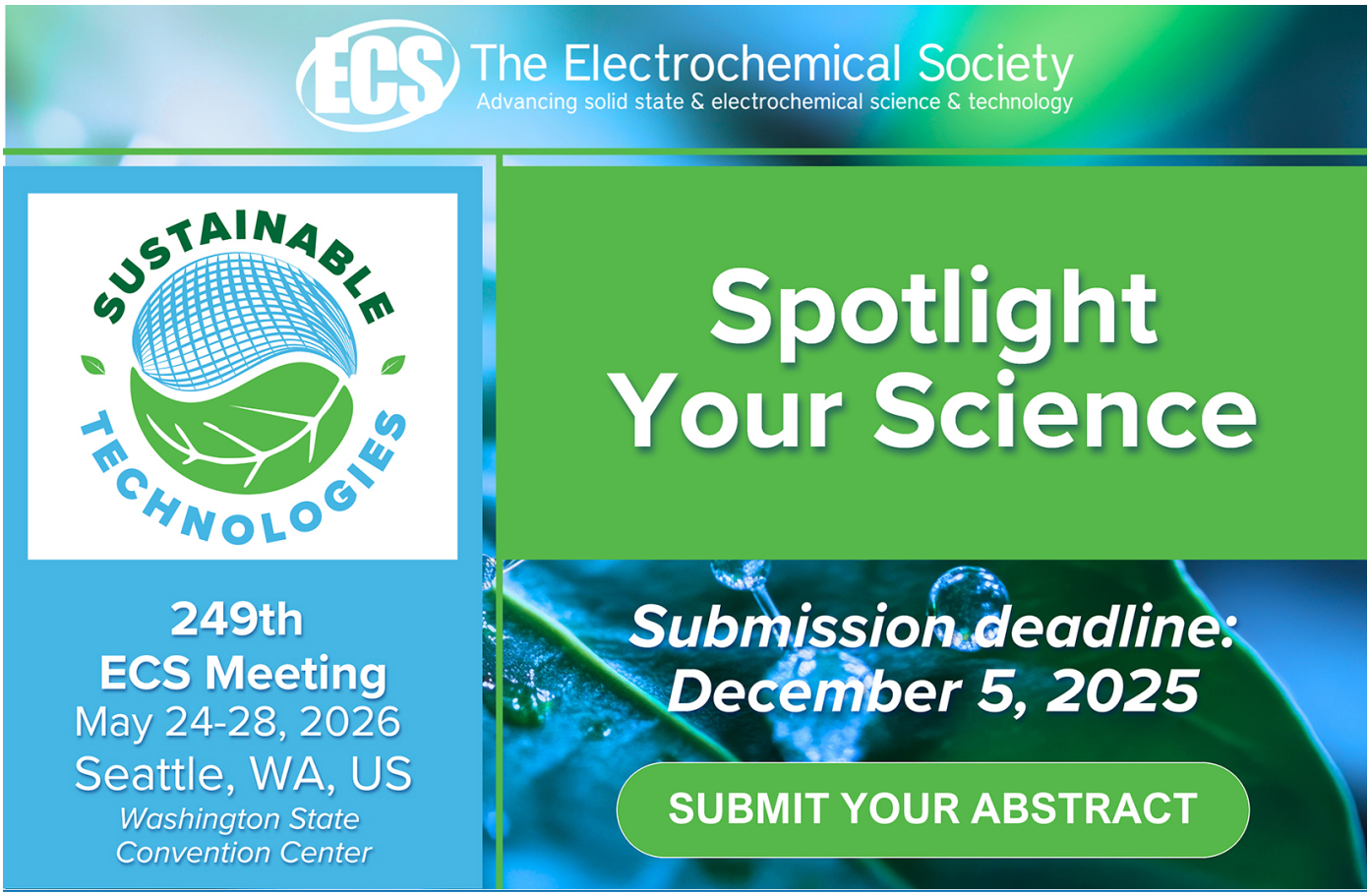
Applications of linear algebra to the study of mathematical modelling of the physical phenomena of heat conduction by electricity

To cite this article: C Nolasco Serna *et al* 2021 *J. Phys.: Conf. Ser.* **1981** 012001

View the article online for updates and enhancements.

You may also like

- Level of development of mathematical logical thinking in the students of the agricultural areas of the Universidad Francisco de Paula Santander, Ocaña, Colombia
C L Garcia Quintero and C M Durán Chinchilla
- Applications of the Poisson and diffusion equations to materials science
C Nolasco, N J Jácome and N A Hurtado-Lugo
- Analysis of the biomass produced in the furniture factories of the municipality of Ocaña
J Arévalo, E E Espinel and M I Cabellos



This content was downloaded from IP address 8.243.64.131 on 28/10/2025 at 15:47

Journal of Physics: Conference Series **1981** (2021) 012001 doi:10.1088/1742-6596/1981/1/012001

Applications of linear algebra to the study of mathematical modelling of the physical phenomena of heat conduction by electricity

C Nolasco Serna[1], **N Afanador Garcia**[2], **and G Guerrero Gómez**[3]
[1] Grupo de Investigación de la Facultad de Educación, Artes y Humanidades Universidad Francisco de Paula Santander, Seccional Ocaña, Colombia
[2] Grupo de Investigación en Ingeniería Civil, Universidad Francisco de Paula Santander, Seccional Ocaña, Colombia
[3] Grupo de Investigación de Tecnología y Desarrollo en Ingeniería, Universidad Francisco de Paula Santander, Seccional Ocaña, Colombia

E-mail: cnolascos@ufpso.edu.co

Abstract. The study of the phenomenon of electrical conduction has its origin in two historical antecedents, Fourier's law and Maxwell's equations**.** The mathematical formulation of electrical conduction has been extensively studied and the differential equations describing the phenomenon are known. The mathematical solution of the physical model of electrical conduction employs different techniques, the best known of which are the Fourier series, Grenn functions and Bessel equations. The purpose of this research is to present a model of heat conduction with the use of electric current that dissipates heat by convection. The research proposes a method for solving the mathematical model associated with the conduction phenomenon using linear algebra. The advantage of using linear algebra will allow to establish a step-by-step procedure that could be used to study phenomena related to heat conduction, in addition to allowing its implementation through programming. In order to establish the fit of the method derived from linear algebra, the analytical solution and the solution proposed in the research were compared to verify that the proposed method fits with a small error.

1. Introduction

The modeling of the laws of physics through the use of ordinary or partial differential equations has an extensive use. The mathematical formulation of Newton's law [1] and its subsequent extension to analytical mechanics is a remarkable example of the use of differential equations that today show an important impact on engineering applications. The experimental development of the physical laws governing electric and magnetic phenomena and the subsequent mathematical formulation of Maxwell's equations [2] are a remarkable example of the use of partial differential equations to understand physical phenomena.

The study of physical phenomena involving heat conduction and electrical effects has been widely developed and the associated mathematical modeling is an area of great research activity [3]. In particular, the study of analytical methods to study the solution of the heat conduction model by means of an electrical source is a well-known topic [4-6]. The present research is relevant in the first place because it proposes a mathematical model through the use of ordinary differential equations that allows describing heat conduction in a metallic plate through the use of electrity, with the novelty of including

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd

the convective effect in the process [7]. Secondly, we propose a method for solving the mathematical model of heat conduction with the advantage of using known concepts of calculus in several variables and linear algebra, as well as facilitating its use through programming. Thirdly, the numerical solution describes the temperature function with a degree of certainty that is quite faithful to the analytical solution. Finally, the methodology for solving the differential equation can be extended by researchers in related areas as well as serving as a tool in teaching applications of linear algebra methods to solve engineering problems.

This research is part of a group of research related to the modeling of energetic phenomena carried out by professors in the field of energy modeling in the mechanical engineering department of Universidad Francisco de Paula Santander, Ocaña, Colombia [8-10].

2. Mathematical modeling of one-dimensional heat diffusion and convective system

The modeling of a physical phenomenon from a mathematical point of view must be described in terms of a set of differential equations that describe the variables involved. This section presents the mathematical model that describes the process of heat conduction by the use of electricity in the presence of the convection effect. The methodology for the search of the solution through the use of linear algebra is then proposed.

2.1. Formulation of the mathematical model

The Equation (1) describes the one-dimensional heat conduction process in a solid steel bar as a particular case of the energy equation associated with the heat transfer process [11].

$$\frac{d^2T}{dR^2} + \frac{q}{R}\frac{\partial T}{dR} + \frac{1}{k}f(R) = 0,\ R \neq 0. \tag{1}$$

In the case that the geometry of the bar is a cylinder or a sphere, applying L'Hospital's rule in the case $R = 0$ generates the Equation (2).

$$(1+q)\frac{d^2T}{dR^2} + \frac{1}{k}f(R) = 0,\ R = 0. \tag{2}$$

When the bar has a length L, the boundary conditions are represented in Equation (3).

$$T(0) = 0,\ T(L) = T_L. \tag{3}$$

The convective effect [12] at $R = L$ at an ambient temperature T_a with a transfer coefficient g_L is given by Equation (4).

$$k\frac{dT(R)}{dR} + g_N T(R) = g_N T_a \quad \text{at } R = L. \tag{4}$$

The complete Equations (1) to Equation (4) represent the mathematical modeling of the heat transfer with electric current which generate energy by convection in a solid steel bar.

2.2. Linear algebra method for the heat conduction model

To generate approximations of the second derivative, the Taylor formula is used, which is represented in Equation (5).

$$T(x+E) = f(x) + f'(x)E + \cdots + \frac{1}{(n+1)!}f^{(n+1)}(x)E^{n+1}. \tag{5}$$

Using the Equation (5) to calculate the second derivative of the temperature generates the Equation (6).

$$T_j''(t) = \frac{1}{(\Delta R)^2}(T_{j-1}(t) - 2T_j(t) + T_{j+1}(t). \quad (6)$$

Substituting complete Equations (6) in Equation (1) we have complete Equation (7).

$$\left(1 - \frac{q}{2j}\right)T_{j-1} - 2T_j + \left(1 + \frac{q}{2j}\right)T_{j+1} + \frac{(\Delta R)^2}{k}f_j = 0, \quad (7)$$

where $j = 1,2, \dots, N-1$. When $R = 0$, the finite difference for Equation (2) is determined as Equation (8).

$$2(1+q)(T_1 - T_0) + \frac{(\Delta R)^2}{k}f_0 = 0, \quad j = 0. \quad (8)$$

In the case that $j = N$, and eliminating the variable T_{N+1} in the complete Equation (4) and Equation (7) generates the complete Equation (9).

$$2T_{N-1} - 2\alpha_N T_N + 2\beta_N + G_N = 0, \quad j = N. \quad (9)$$

The parameters in Equation (9) are determinated by the complete Equations (10) to Equation (12).

$$\alpha_N = 1 + (1 + \frac{q}{2N})\frac{\Delta x g_N}{k}, \quad (10)$$

$$\beta_N = \left(1 + \frac{q}{2N}\right)\frac{\Delta x g_N T_a}{k}, \quad (11)$$

$$G_N = \frac{(\Delta x)^2 g_N}{k}. \quad (12)$$

The complete Equation (7), Equation (8) and Equation (9) provides $N+1$ relations for the the determination of $N+1$ unknown node temperatures determinated by the matricial system $AX + b$, where the matrix A and the columns vectors X and b are defined by the complete Equations (13) to Equation (15) in the case of $N = 3$.

$$A = \begin{pmatrix} -2(1+q) & 2(1+q) & 0 \\ 1 - \frac{q}{2} & -2 & 1 + \frac{q}{2} \\ 0 & 2 & -2\alpha_2 \end{pmatrix}, \quad (13)$$

$$X = \begin{pmatrix} T_0 \\ T_1 \\ T_2 \end{pmatrix}, \quad (14)$$

$$b = \begin{pmatrix} \frac{f_0(\Delta x)^2}{k} \\ \frac{f_1(\Delta x)^2}{k} \\ 2\beta_2 + G_2 \end{pmatrix}. \quad (15)$$

3. Results and discussion

In order to verify the solution of the previous section for the mathematical model of conduction, we consider the heating by an electric current of a metal plate with circular shape that dissipates heat on the surface by convection. The parameters of the physical phenomena are $R = 0.8$ cm, $N = 8$, $q = 2$, the

thermal conductivity constant $\mathrm{k} = 40\ \frac{\mathrm{w}}{\mathrm{m}}$℃, the energy generation rate $\mathrm{f} = 2000000\ \frac{\mathrm{w}}{\mathrm{m}^3}$, the heat transfer coefficient $\mathrm{g_N} = 400\frac{\mathrm{w}}{\mathrm{m}^2}$ ℃ and the ambient temperature $\mathrm{T_a} = 30$ ℃. Substituting the above values into the complete Equation (7), Equation (8) and Equation (9) gives the system of Equations (16) to Equation (17).

$$6(\mathrm{T_1} - \mathrm{T_0}) + 5 = 0, \tag{16}$$

$$\left(1 - \frac{1}{j}\right)T_{j-1} - 2T_i + \left(1 + \frac{1}{j}\right)T_{j+1} + 5 = 0, \tag{17}$$

$$T_7 - 1.1125T_8 + 5.875 = 0. \tag{18}$$

The previous system of equations can be represented in its matrix form $AX + b = 0$. The exact solution [13] of the complete Equations (1) to Equation (4) are defined in Equation (19).

$$T(R) = \frac{490}{3} + \frac{160}{3}\left[1 - \left(\frac{R}{0.08}\right)^2\right]. \tag{19}$$

The Table 1 shows the results of the plate temperature for different positions from the center of the plate. The results of the exact solution, Equation (19), and the solution of the system of Equations (16) to Equation (18) are the same compared.

Figure 1 shows a comparison of the interpolation plot of the data in Table 1 and the exact solution of the mathematical model of heat conduction by electric current.

Table 1. Linear algebra solution of conduction model.

$\frac{R}{L}$	Exact and linear algebra solution
0	216.6667
0.1	215.8333
0.2	213.3333
0.3	209.1667
0.4	203.3333
0.5	195.8333
0.6	186.6667
0.7	175.8333
0.8	163.3333

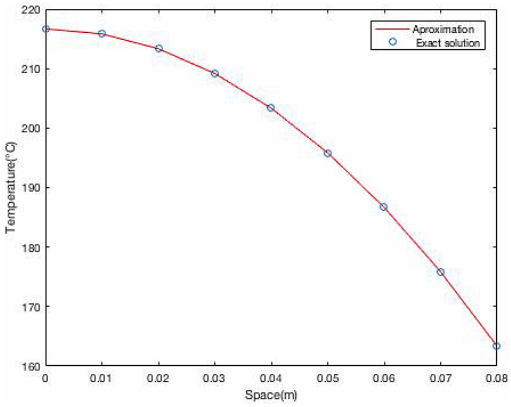


Figure 1. Contour of temperature with the method of linear algebra and exact solution.

From Table 1 and Figure 1 it is possible to conclude that the method proposed in the present research, which is derived from linear algebra tools, fits very well in comparison with the exact solution of the mathematical model presented in the Equations (1) to Equation (4). The proposed method for solving the heat conduction model is suitable to be applied to physical phenomena with similar mathematical modeling. A set of mathematical models suitable for applying the solution method described in the research will be listed below. A transient heat conduction in a fin [15] is described by de mathematical model in Equation (20).

$$\frac{\partial^2 \mathrm{T}}{\partial \mathrm{x}^2} - \mathrm{m}^2(\mathrm{T} - \mathrm{T_a}) = \frac{1}{\alpha}\frac{\partial \mathrm{T}}{\partial \mathrm{t}}. \tag{20}$$

The equation governing heat transfer in liquid crystals [16] is the Equation (21).

$$\frac{\partial^2 T}{\partial x^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{1}{D}\frac{\partial T}{\partial t} = \frac{-Q(t)}{K}. \quad (21)$$

The experiment to measure the process of heat flow in two dimensions [17] are described by the Equation (22).

$$C_p \frac{\partial T}{\partial t} - k\nabla^2(T) = 0. \quad (22)$$

Differential Equations (20) toEquation (22) describe physical phenomena in which it is possible to apply the tools of linear algebra developed in the research.

4. Conclusion

The methodology to solve the heat conduction model with convective effects consists of substituting the first and second order derivatives by their equivalent Taylor approximation. The effect of substituting the approximations in the mathematical model resulted in the generation of a system of equations that can be solved by linear algebra methods. This approach had the positive effect of fitting the exact solution very well. We propose that the above methodology can be applied to different differential equations.

References

[1] Kleppner D, Kolenkow R J 1978 *An Introduction to Mechanics* (Singapore: McGraw-Hill)
[2] Marsden J E, Tromba A J 1991 *Cálculo Vectorial* (Delawere: Addison-Wesley)
[3] Baedr H D, Stephan K 1998 *Heat and Mass Transfer* (Berlin: Springer-Verlag)
[4] Tirabassi T, Buske D, Moreira D, Vilhena M 2008 A two-dimensional solution of the advection-diffusion equation with deposition to the ground *Journal of Applied Meteorology and Climatology* **47** 2096
[5] Moitsheki R, Rowjee A 2011 Steady heat transfer through a two-dimensional rectangular straight fin *Mathematical Problems in Engineering* **2011** 1
[6] Akbarzadeh A, Chen Z 2012 Heat conduction in one-dimensional functionally graded media based on dual-phase-lag theory *Journal of Mechanical Engineering Science* **227(4)** 1
[7] Ozisik M N 1993 *Heat Conduction* (New York: John Wiley)
[8] Guerrero Gómez G, Espinel Blanco E, Sánchez Acevedo H G 2017 Análisis de temperaturas durante la cocción de ladrillos macizos y sus propiedades finales *Revista Tecnura* **21(51)** 118
[9] Nolasco C, Guerrero Gómez G, Gómez J A 2019 Mathematical model of firing process of Ladrillera Ocaña, Colombia *Journal of Physics: Conference Series* **1408** 012017:1
[10] Nolasco C, Jacome N J, Hurtado N A 2019 Solution by numerical methods of the heat equation in engineering applications Mathematical A case of study: Cooling without the use of electricity *Jornal of Physics: Conference Series* **1388** 012017:1
[11] Poirier D R, Geiger G H 2016 *Transport Phenomena in Materials Processing* (Berlin: Springer-Verlag)
[12] Bergman T, Lavine A, Incropera F, Dewitt D 2011 *Introduction to Heat Transfer* (New York: Jonh Wiley and Sons)
[13] Carslaw H S, Jaeger J C 1959 *Conduction of Heat in Solids* (New York: Oxford University Press)
[14] Burden R, Faires J 2010 *Análisis Numérico* (México: Editorial Cengage Learning)
[15] Brody J, Brown M 2017 Transient heat conduction in a heat fin *American Journal of Physics* **85** 582
[16] Bacon E, Wick R, Hecking P 1995 Heat, light, and videotapes: Experiments in heat conduction using liquid crystal film *American Journal of Physics* **63** 359
[17] McDougall P, Ayars E 2014 Two-dimensional heat flow apparatus *American Journal of Physics* **82** 620