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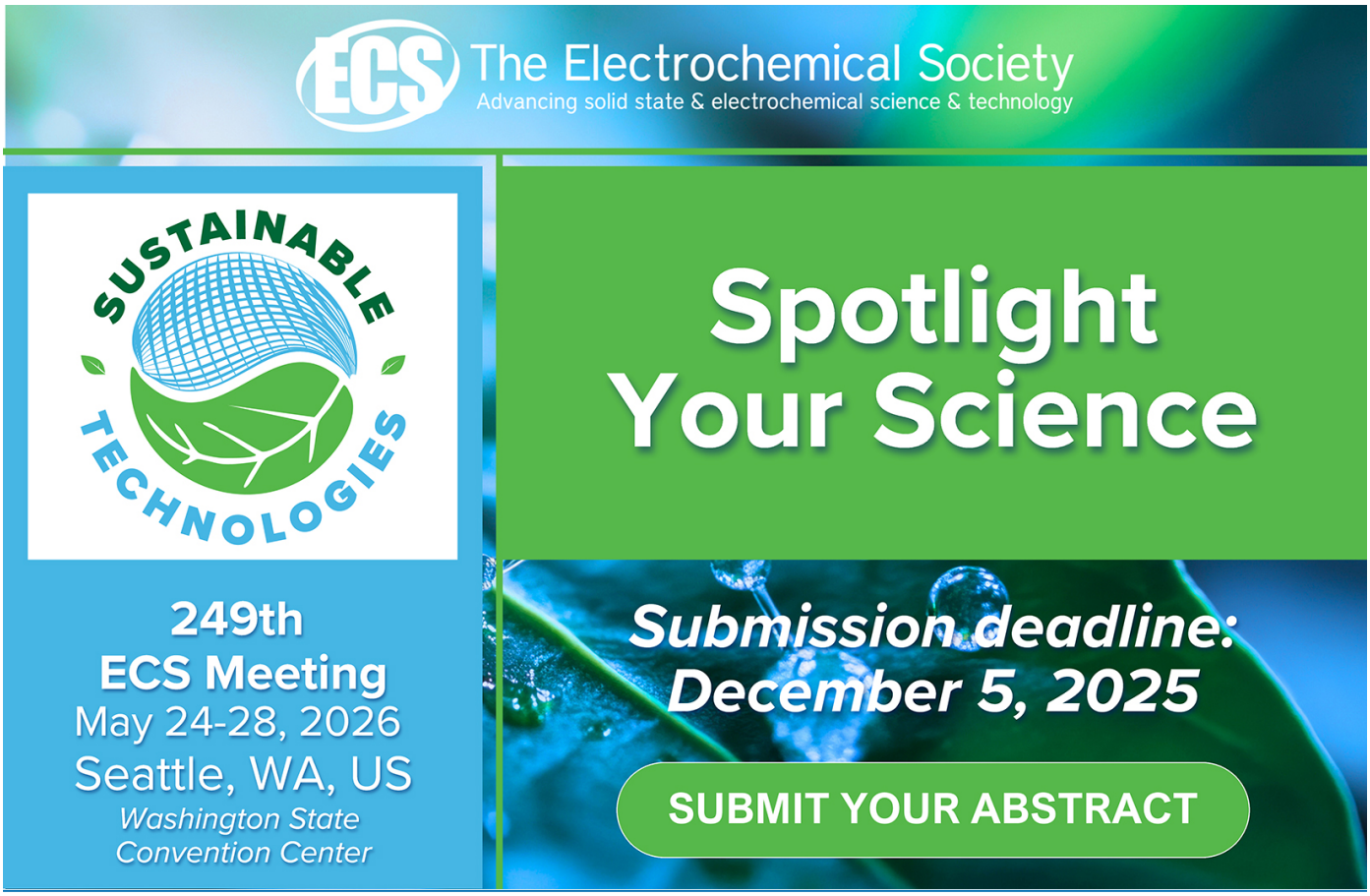
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Quantification of uncertainty in steel plates subject to fatigue with variable load via chaos expansion polynomial

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Abstract. This investigation analyzed the propagation of the uncertainty of the cycle number to fatigue failure, taking into account the strain life method, considering as random variables some physical parameters of the material and load vector in a hole-in-center; plate composed of an aluminum alloy 2024-T351 under fatigue, with variable amplitude loading through multidimensional Hermite polynomials. The application of series of multidimensional Hermite polynomials allowed the prediction of the randomness of the output vector. This research demonstrated that a 2nd degree multidimensional Hermite polynomial adequately estimates the propagation of uncertainty with 45 points per random variable, while the use of Monte Carlo simulation requires 1E + 06 points per random variable. This generates significant computational savings and time. The results showed that variations in the material parameters are not important compared to the variations in the load parameters, which generate an increase in the probability of failure in aluminum alloy parts subjected to fatigue.

1. Introduction

The theory and methods for quantifying uncertainty have evolved significantly in the last four decades. The analysis and design using random behavior has been gaining space in the scientific community, since, in problems where the randomness is small, we use deterministic models. The propagation of uncertainty in the model parameters can be evaluated using: (i) the analytical method [1,2], (ii) the Monte Carlo method [3,4]. In the analytical method, the uncertainty in the output is explicitly represented as uncertainty functions in the input variables, and is useful for small values of uncertainty and is not always applicable to complex or non-linear models in the input parameters. The Monte Carlo method involves a sufficiently large number of simulations for the input random variables, estimated by the probability density functions, $(f_{\boldsymbol{X}}(x))$, with great computational cost and time. It is necessary to estimate the uncertainty without loss of precision in any type of model using a significantly reduced number of solutions.

A new technique has been developed with a lower computational cost than that used by the Monte Carlo method, without loss of precision, called chaos expansion polynomial (PCE) [5, 6]. The PCE in a multidimensional Gaussian random space is defined by the Hermite Multidimensional polynomials. The PCE can be classified into two approaches: (i) intrusive

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formulation, (ii) non-intrusive formulation; in the intrusive formulation the uncertainty is explicitly expressed within the analysis of the system under investigation (Galerkin's method [7]). In the non-intrusive formulation, PCEs are used to create response surfaces (called stochastic response surfaces [8]) without interfering with the analysis of the system [9].

The specialized literature does not report any study of the quantification of uncertainty in steel elements subject to fatigue with PCE. This investigation examines the randomness of the properties of the material and the applied load, in the determination of the life cycle, N_f, under the deformation-life methodology using PCE with a non-intrusive approach. The unknown coefficients of the PCE were determined using the probabilistic placement method [10, 11]. The main contribution of this research was the quantification of the propagation of the uncertainty of steel plates subjected to fatigue considering randomness in material properties and in the amplitude of the applied load through the determination of $f_{\boldsymbol{X}}(x)$.

2. Polynomial chaos expansion

Consider a physical model whose uncertainty in the input parameters are modeled as normal independent random variables ($X_1,\ X_2, \ldots,\ X_M$) gathered in a vector $\boldsymbol{X}$, with dimension M. The behavior of the system is described by the function f, deterministic model of the black box type (or mathematical model for this case), see Equation (1).

$$Y = f(\boldsymbol{X}), \tag{1}$$

where, Y denotes the random scalar output; without loss of generality, we can assume that the input is distributed normally with average (μ o $E[Y]$) and standard deviation (σ). The function f can be decomposed into summands of increasing dimension, see Equation (2) according to [12].

$$f(\boldsymbol{X}) = E[Y] + \sum_{i=1}^{M} f_i(X_i) + \sum_{1 \leqslant i \leqslant j \leqslant M} f_{i,j}(X_1, X_j) + \cdots + f_{1,\ldots,M}(X_1, \ldots, X_M). \tag{2}$$

If the response Y has a finite variation, the f function can be compactly represented according [13], in a Hilbertian polynomial basis, as indicated in Equation (3).

$$f(\boldsymbol{X}) = \sum_{\boldsymbol{\alpha} \,\epsilon\, \mathbb{N}^M} a_{\boldsymbol{\alpha}} \Psi_{\boldsymbol{\alpha}}(\boldsymbol{X}). \tag{3}$$

The Equation (3) is composed of two parts: (i) known and (ii) unknown, (i) the known part corresponds to Ψ_α's, multidimensional polynomials, which involves the product of one-dimensional polynomials evaluated in a standard normal sample space, and (ii) the unknown part is defined by the deterministic coefficients (a_α's), which can be solved using least squares regression in a non-intrusive formulation. In the case of random and independent input variables they can be transformed from the space of the random input variable to a standard normal space, by means of isoprobabilistic transformations, so $\boldsymbol{X}$ is represented by a standard normal vector $\boldsymbol{\xi} = (\xi_1,\ \xi_2,\ \ldots,\ \xi_m)$. For computational reasons, without loss of precision the series must be truncated, only polynomials of degree p or less are adopted. The complexity of the model determines the degree of the truncated order polynomial p, according to [1], see Equation (4).

$$P = \frac{(m+p)!}{m!\,p!}, \tag{4}$$

where P is the total number of terms of a complete chaos expansion polynomial of order p for a response function Y, involving n random input variables.

Once the basis is defined and the coefficients determined, the expansion can be use as a surrogate model for the high-fidelity model.

3. Notched bodies under variable amplitude loading

In real service conditions, a structure may be subjected to different load ranges as shown in Figure 1.

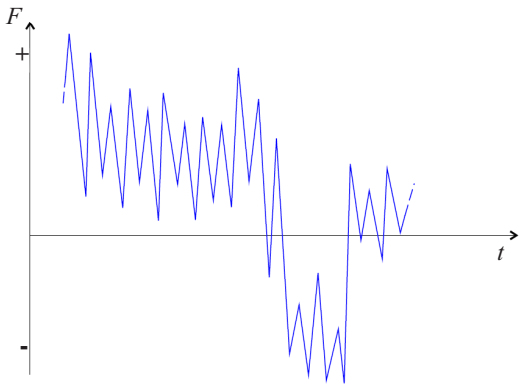


Figure 1. Variable amplitude loading.

The loads of variable amplitude can be in tension, compression or in tension-compression with or without medium tension. In Figure 1 most of the loads are in tension without the effect of mean stress. The variable amplitude loading can be analyzed using the Palmgren-Miner rule [15, 16], which assesses the damage caused by each tension block S_a see Figure 2(a), and integrated material failure, which occurs when the sum of the damage is equal to 1. The damage is defined as the relationship between the N_1 amplitude loading cycles S_{a1}, damage produced can be defined $D_1 = \frac{N_1}{N_{f1}}$, where N_{f1} is the number of cycles to generate fatigue failure due to amplitude loading S_{a1}, see Equation (5).

$$\sum_{i=1}^{N} D_i = \frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \cdots + \frac{N_n}{N_{fn}} = 1 \tag{5}$$

Each load cycle (S_{ai}) defines the number of cycles to fatigue failure (N_{fi}) see Figure 2(a), that is, the sum of the pairs of points (S_{ai}, N_{fi}) for each load cycle establishes the $S-N$ curve, which relations the amplitude of tensions and the number of cycles for fatigue failure, see Figure 2(b).

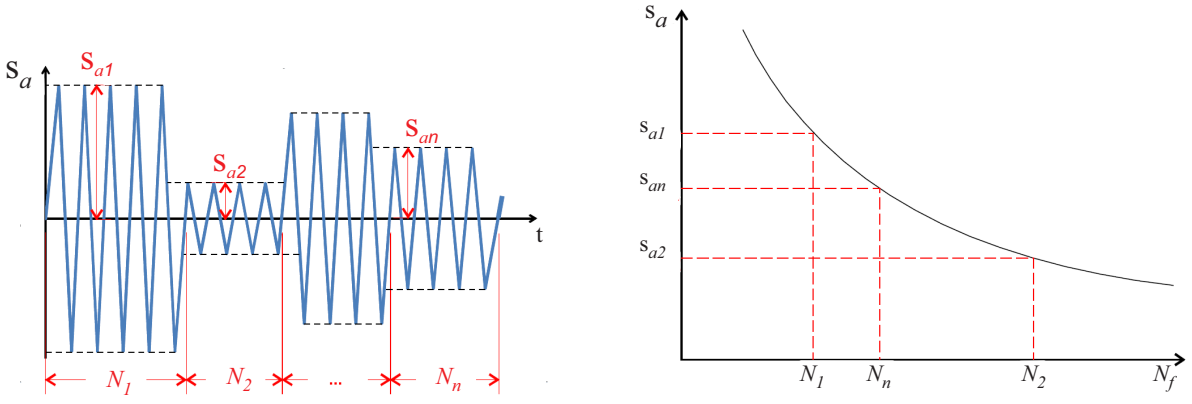


Figure 2. Life prediction, according to the Palmgren-Miner rule, (a) variable amplitude loading formed by constant amplitude sections, (b) S-N curve. Adapted from [17]

.

The methodology used to calculate the N_f of notched components by life-deformation requires knowledge of applied stress (S_t) and deformation (ε) that act on the root of the notch, these S_t and ε can be obtained by finite element or approximate methods. The S_t and ε at the root of the notch were estimated by elastic analysis. The method Neuber [18] was used as an approximate method to estimate the S_t and ε at the root of the notch.

In the Equation (6) defined the relation of stress and deformation for perforated element under monotonic loading, where $S_t \leqslant S_0$ (S_0 yield stress) hold that.

$$K_s = \frac{S_t}{S}; \;\; K_e = \frac{\varepsilon}{e}; \tag{6}$$

where K_s is the level of stress concentration, S is the nominal stress, K_e is the level of deformation concentration and e is the nominal strain defined as the strain corresponding to S in the S_t - ε curve. The Neuber method [18] establishes that the elastic stress concentration factor (K_t) is equal to the geometric average between the K_S e K_ε, see Equation (7).

$$K_t = \sqrt{K_S K_\varepsilon}. \tag{7}$$

Replacing the Equation (6) in Equation (7) and considering an elastic behavior ($S = Ee$), the Equation (8) is obtained.

$$S_t \varepsilon = \frac{(K_t S)^2}{E}. \tag{8}$$

At the root of the notch, the stresses and strains must follow the stress-strain ratio of the material, so for the Ramberg-Osgood ratio, see Equation (9).

$$\varepsilon = \frac{S_t}{E} + \left(\frac{S_t}{H'}\right)^{\frac{1}{n'}}, \tag{9}$$

where H' and n' are the hardness coefficient and the monotonic hardening exponent, respectively. Substituting the Equation (9) in Equation (8), get the Equation (10) nonlinear, which provides the S_t and ε at the root of the notch, whose solution is the stress at the root of the notch. Known to S_t, it is replaced in Equation (9) to get ε at the root of the hole notch.

$$\frac{S_t^2}{E} + S_t \left(\frac{S_t}{H'}\right)^{\frac{1}{n'}} = \frac{(K_t S)^2}{E}. \tag{10}$$

4. Numerical implementation

A numerical model was implemented using a MatLab programming environment to establish the number of cycles to fatigue failure (N_f) implementing the strain-life and PEC methodology. The results were validated using Monte Carlo simulation (MCS) on notched bodies subject to fatigue with variable amplitude loading.

4.1. Failure modeling in notched bodies under variable amplitude loading

In the implementation of the strain-life methodology, the Neuber method [18] and the Morrow relationship [19] were used to estimate the number of cycles to fatigue failure (N_f), see Equation (11).

$$\varepsilon_a = \frac{S'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c. \tag{11}$$

Probabilistic analyzes of the N_f, considering material uncertainty (H', n', S'_f e ϵ'_f), where S'_f fatigue resistance coefficient and ϵ'_f fatigue ductility coefficient, and variable amplitude loading average ($\mu_{\boldsymbol{S0}}$) was made, is illustrated in Figure 3(a) for a hole-in-center plate composed of an aluminum alloy 2024-T351 as shown in Figure 3(b).

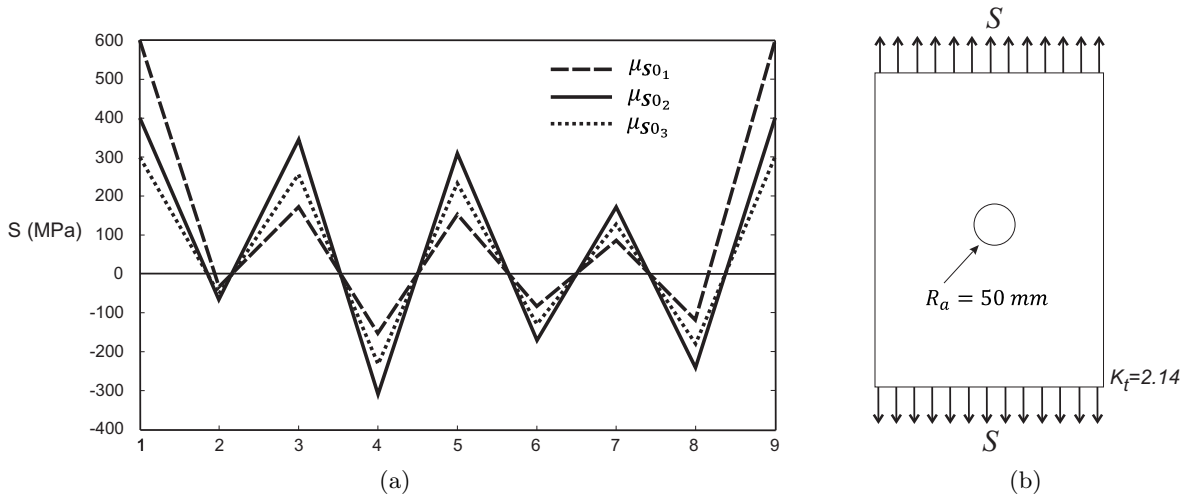


Figure 3. Fatigue case study with variable amplitude loading, (a) hole-in-center plate, and (b) variable amplitude loading averages, $\mu_{\boldsymbol{S0}}$.

The deterministic and probabilistic parameters of the variables of the material and the amplitude of load are shown in the Table 1, where μ is the average value, V is the coefficient of variation of the random variables and $f_{\boldsymbol{X}}(x)$ the probability density function of $\boldsymbol{X}$.

Table 1. Random and deterministic variables of the variable amplitude nominal stress history applied to the aluminum alloy plate 2024-T351 with center hole.

Variables	μ	V (%)	$f_{\boldsymbol{X}}(x)$	Variable (MPa)	$\mu_{\boldsymbol{S0}_i}$ i=1	i=2	i=3	V (%)	$f_{\boldsymbol{X}}(x)$
H' (MPa)	662.00	15	$\mathcal{N}$	$S0_i(1)$	600.00	400.00	300.00	20	$\mathcal{N}$
n' (-)	0.07	15	$\mathcal{N}$	$S0_i(2)$	-35.00	-69.00	-51.75	20	$\mathcal{N}$
σ'_f (MPa)	927.00	15	$\mathcal{N}$	$S0_i(3)$	173.00	345.00	258.75	20	$\mathcal{N}$
ϵ'_f (-)	0.41	15	$\mathcal{N}$	$S0_i(4)$	-155.00	-310.00	-232.50	20	$\mathcal{N}$
E (MPa)	73100.00	0	‡	$S0_i(5)$	155.00	310.00	232.50	0	‡
b (-)	-0.11	0	‡	$S0_i(6)$	-86.00	-172.00	-129.00	0	‡
c (-)	-0.71	0	‡	$S0_i(7)$	86.00	172.00	129.00	0	‡
K_f (-)	2.40	0	‡	$S0_i(8)$	-121.00	-241.00	-180.75	0	‡

(-) Dimensionless $\mathcal{N}$ Normal ‡ Deterministic without $f_{\boldsymbol{X}}(x)$; $\mu_{\boldsymbol{S0}_i}$ (MPa)

Given the probability density function of the number of cycles for fatigue failure, $f_{\hat{\boldsymbol{N}}_f}(\hat{N}_f)$ (using full PEC, PEC_C) and $f_{\boldsymbol{N}_f}(N_f)$ (using MCS) which best defines data behavior using PEC_C and MCS, and different types of error were determined to evaluate the results, are shown in Figure 4. Figure 4(a) shows the results for $f_{\hat{\boldsymbol{N}}_f}(\hat{N}_f)$ and $f_{\boldsymbol{N}_f}(N_f)$ para $S0_1$ as well as the error obtained, while Figure 4(b) corresponds to $S0_2$ and finally the results found for $S0_3$ are

represented in Figure 4(c). In Figure 4, MSE corresponds to mean square error, NRMSE is the standardized root of the mean square error and NMSE is the standardized mean square error. The errors that accompany the $f_{\boldsymbol{X}}(x)$ determined using the maximum likelihood estimator, indicate that the results found using PEC_C with grade 2 (5E+04 simulations) are similar to those found using MCS with 1E+06 simulations.

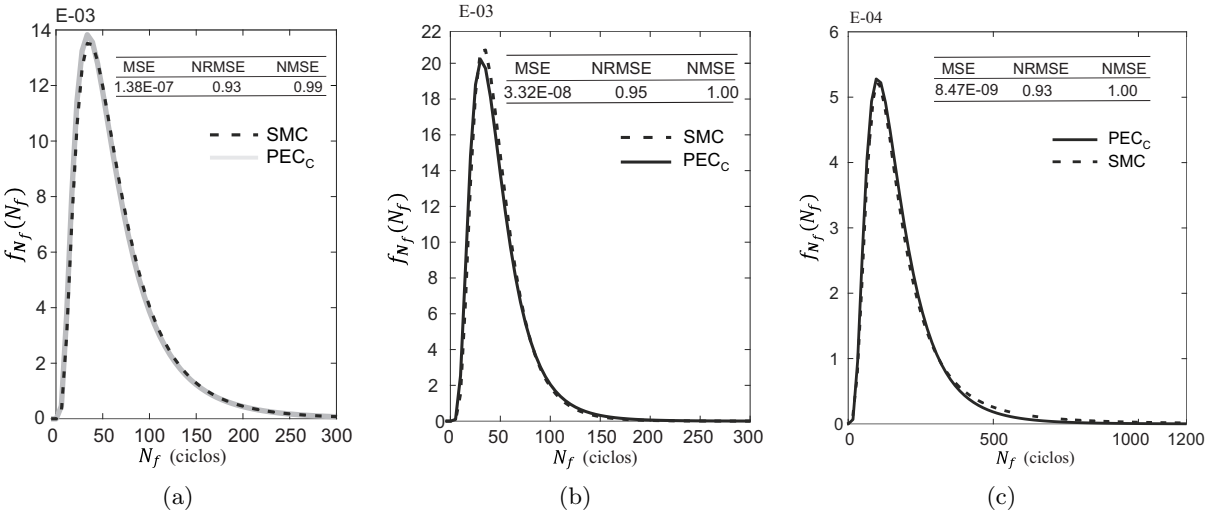


Figure 4. Comparison of $f_{\hat{\boldsymbol{N}}f}(\hat{N}_f)$ and $f_{\boldsymbol{N}f}(N_f)$ of the PEC and MCS for plate with center hole with different $S0$, (a) $S0_1$, (b) $S0_2$ and (c) $S0_3$.

The behavior of $f_{\hat{\boldsymbol{N}}\boldsymbol{f}}(\hat{N}_f)$ and the $f_{\boldsymbol{N}\boldsymbol{f}}(N_f)$ are similar, as are the values of the $\mu_{\hat{\boldsymbol{N}}\boldsymbol{f}}$ and $\mu_{\boldsymbol{N}\boldsymbol{f}}$, same as the V the results obtained are classified as low cycle fatigue [20–24]. The $f_{\boldsymbol{N}\boldsymbol{f}}(N_f)$ corresponds to lognormal for both PEC and MCS, very different from that observed in almost fragile materials [25].

5. Conclusions

The PEC adequately estimates the $f_{\hat{\boldsymbol{N}}f}(\hat{N}_f)$ using the life-deformation methodology with respect to the results obtained using MCS, with a lower number of simulations, generating a decrease in computational cost. The PEC estimates with a small mean square error (MSE) of the order E-07 and allows to determine the probability of fatigue failure with few simulations. The PEC methodology established a relationship between the output variable and the eight random input variables, allowing to estimate the value of $\hat{N}_f$ for different combinations of the input variables. This methodology makes it possible to relate the random variables of the material and the load amplitude with respect to the physical property that the material has to fatigue, that is, the number of cycles for fatigue failure.

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