Journal of Physics:
Conference Series

PURPOSE-LED PUBLISHING™

PAPER • **OPEN ACCESS**

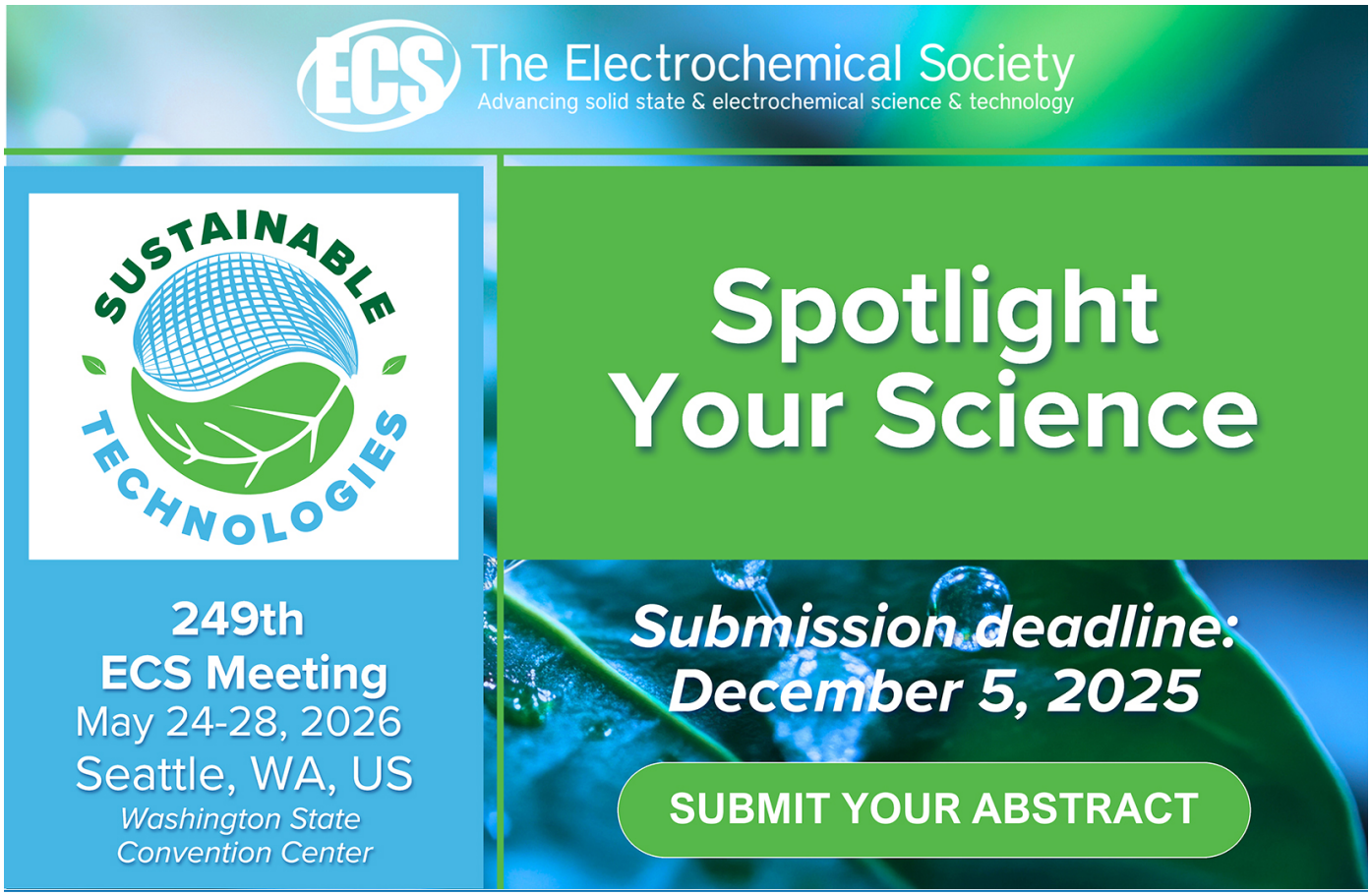
Mathematical model of physical process of heat transfer in liquid crystal

To cite this article: C Nolasco *et al* 2020 *J. Phys.: Conf. Ser.* **1587** 012013

View the article online for updates and enhancements.

You may also like

- Applications of the Poisson and diffusion equations to materials science
C Nolasco, N J Jácome and N A Hurtado-Lugo
- Analysis of the biomass produced in the furniture factories of the municipality of Ocaña
J Arévalo, E E Espinel and M I Cabellos
- Mathematical model of the firing process in Ladrillera Ocaña, Colombia
C Nolasco, G Gurrero Gómez Jácome and J A Gómez Campero



This content was downloaded from IP address 190.90.37.62 on 15/10/2025 at 22:33

Mathematical model of physical process of heat transfer in liquid crystal

C Nolasco[1], B M Velascos Burgos[2], and J J Cadena Morales[1]
[1] Grupo de Investigación de la Facultad de Educación, Artes y Humanidades, Universidad Francisco de Paula Santander, Seccional Ocaña, Colombia
[2] Grupo de Investigación de la Facultad de Ciencias Administrativas, Universidad Francisco de Paula Santander, Seccional Ocaña, Colombia

E-mail: cnolascos@ufpso.edu.co

Abstract. The process of heat transfer by conduction is relevant in different context of engineering and sciences. An example of this situation is confirmed by the study of the thermal properties of liquid crystal sheets to measure the effects of heat transfer on metal surfaces. The aim of this work is to establish a mathematical model by numerical methods to determine the physical process of heat transfer in liquid crystal. For the data acquisition, an experiment that measures thermal properties on an aluminum plate was designed. We proceed to propose a mathematical model that uses the Laplace equation. Also, we proceed to calculate the solution of the equation by the finite difference method and then make a comparison with the analytical solution and the experimental data.

1. Introduction

In the development plan of Norte de Santander, Colombia, it is contemplated [1] the investigation of different areas of interest such as the energy mining, agribusiness and manufacturing sectors. In each of these sectors, research in materials science is of vital importance. In the region of Norte de Santander in recent investigations the processes of heat transfer on variety of materials have been studied, for a study of this situation [2-4] they are a valuable contribution in this direction. This research opens a different line of work when considering the study of heat transfer in liquid crystals.

Austrian botanist Friedrich Reintzer, interested in the biological function of cholesterol in plants, began observing the dissolution of a certain substance related to cholesterol. He noted that the dissolved substance had an opaque appearance at 145.5 ºC, by increasing the temperature to 178.5 ºC, the substance takes on the appearance of in a clear liquid. When applying the cooling process to the liquid, the appearance of an unplanned blue color in the temperature transition spectrum and a violet blue color just before crystallization was observed. This type of phenomenon led scientists to propose another state of matter called liquid crystal [5].

The thermal properties of liquid crystals have been studied in-depth, for a recount of their physical properties we refer readers to the reference [6]. The thermal properties of liquid crystals can be used to manufacture microspheres and study convection in fluids [7], they can also be used to manufacture thermal control systems. The study of mathematical models with numerical method to explain heat transfer in the context of liquid crystals is little known. The main contribution of this work is to show the simplicity of the numerical method used to model the transference phenomenon in contrast to the

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd

analytical method that usually appears in the specialized works [8]. The proposed numerical method will allow the understanding of heat conduction with the help of a computer program.

The work is organized as follows. In the first part, the description of an experimental assembly will be made to measure the thermal effects in an aluminum foil, in the transfer process it will be necessary to use the liquid crystals. In the second part, the mathematical model is presented in the form of the Laplace equation and then proceeds to calculate the numerical and analytical solution. In the third, the results of the research are presented, and the numerical and experimental results are contrasted.

2. Methodology

The experimental design [8] consists of a tank that contains constant temperature water and a pump. In the upper part of the tank, an aluminum plate is placed a quarter of an inch wide and 10 larges high in good thermal contact with a heat source regulated by a Variac. The plate is covered, in the front and the back, with a glass plate four inches wide. The purpose of glass plates is to reduce the temperature of the faces of the sheet compared to the edges. A sheet of a heat-sensitive liquid crystal is placed between the front of the plate and the glass plate. A thermostatic bath is started, and the heat is monitored by thermometers. The temperature of the liquid crystal sheet varies between 29 ºC - 35 ºC, therefore, the three sides of the sheet is maintained at 29 ºC, the fourth side stays at 35 ºC.

The data acquisition system for the aluminum plate was designed, following the technique of recollecting data in [8] to record the temperatures monitored by thermometers. The method to establish the mathematical model consisted in modeling the temperature through the use of the Laplace equation.

3. Mathematical model

Assuming the material is homogeneous and K, ρ, c are independent of position r̃ y si K, ρ, c are assumed independent of the temperature function T then Equation (1) is the generalized Laplace equation [9].

$$\nabla^2 T - \left(\frac{1}{D}\right)\left(\frac{\partial T}{\partial t}\right) = \frac{-V(\tilde{r},t)}{K}. \tag{1}$$

Here, $D = K/\rho c$ it is the constant of thermal diffusion. If the functionV is independent of time and the state is stable, Equation (1) is reduced to an Equation (2).

$$\nabla^2 T = \frac{-V(\tilde{r},t)}{K}. \tag{2}$$

In the regions where $V(\tilde{r}, t) = 0$ Equation (3) is generated, the well-known Laplace equation.

$$\nabla^2 T = 0. \tag{3}$$

3.1. Analytical solution of the Laplace equation

Equation (4) and Equation (5) summarize the theoretical model for heat transfer in aluminum foil.

$$T_{xx} + T_{yy} = 0, \tag{4}$$

$$T(x,0) = 35,\ T(10,y) = T(x,10) = T(0,y) = 29. \tag{5}$$

The Equation (6) shows the analytical solution of the theoretical model of heat transfer.

$$T(x,y) = T_1 + \sum_{\text{odd } n}^{\infty} c_n \sin\frac{n\pi x}{10} \sinh n\pi \left(1 - \frac{y}{10}\right). \tag{6}$$

Where Fourier coefficients are in the Equation (7).

$$c_n = \frac{4(T_2 - T_1)}{n\pi \sinh n\pi},\ n = 1,3,5,\ldots \tag{7}$$

3.2. Numerical solution of the Laplace equation

To solve the model of Equation (4) and Equation (5) by numerical methods it is necessary to calculate approximations for second-order derivatives. When using the Taylor series [10] shown in the Equation (8).

$$f(x + \Delta x) = f(x) + \Delta x f'(x) + \frac{(\Delta x)^2}{2!} f''(x) + \cdots \quad (8)$$

The Equation (9) is generated.

$$f''(x) = \frac{f(x+\Delta x)+f(x-\Delta x)-2f(x)}{(\Delta x)^2}. \quad (9)$$

By replacing Equation (9) in Equation in (4) the Equation (10) are generated.

$$\frac{T(x+\Delta x,y)+T(x-\Delta x,y)-2T(x,y)}{(\Delta x)^2} + \frac{T(x,y+\Delta y)+T(x,y-\Delta y)-2T(x,y)}{(\Delta y)^2} = 0. \quad (10)$$

If $\Delta x = \Delta y$, the Equation (11) is generated.

$$T(x,y) = 0.25\big(T(x + \Delta x, y) + T(x - \Delta x, y) + T(x, y + \Delta y) + T(x, y - \Delta y)\big). \quad (11)$$

The recursion that is applied to solve Equation (4) is reflected in Equation (12).

$$-4T_{i,j} + T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} = 0. \quad (12)$$

4. Results

The mathematical model to validate the results of the experiment presented in this article has his basis in the mathematical model of the Equation (6). In order to validate the numerical model of Equation (11) and Equation (12), for the physical process of heat transfer for the aluminum plate, we proceed to build a mesh on the aluminum plate and calculate the temperature on each of the nodes. The previous validation is developed through the implementation of a program [11]. Table 1 contains the data used in the program that implements the recursive Equation (12) to solve the Laplace differential equation associated with heat transfer in the aluminum plate. The mathematical model to validate the results of the experiment presented in this article has his basis in the mathematical model of the Equation (6). In order to validate the numerical model of Equation (11) and Equation (12), for the physical process of heat transfer for the aluminum plate, we proceed to build a mesh on the aluminum plate and calculate the temperature on each of the nodes. The previous validation is developed through the implementation of a program [11]. Table 1 contains the data used in the program that implements the recursive Equation (12) to solve the differential Laplace equation associated with heat transfer in the aluminum plate.

Table 1. The parameters used in the design of the code for the explicit method.

Parameters	Data
Plate size	10 cm x 10 cm
Number of points in the x-direction	100
Number of points in the y-direction	100
Temperature in (x, 0)	35 ºC
Temperature in (0, y)	29 ºC
Temperature in (10, y)	29 ºC
Temperature in (x, 10)	29 ºC

The program designed with the data from Table 1 simulates the experiment effect of heat transfer for the aluminum plate shown in the liquid crystal sheets. Figures 1 and Figure 2 show the profile of the temperature that is generated by the program constructing a mesh of size 15 × 15 and 30 × 30 on the aluminum plate and calculating Equation (12) in each of the interior nodes of the mesh. The external nodes of the mesh correspond to the boundary conditions of Equation (5). The design of Figure 1 and Figure 2 represents the contours recorded in the liquid crystal film.

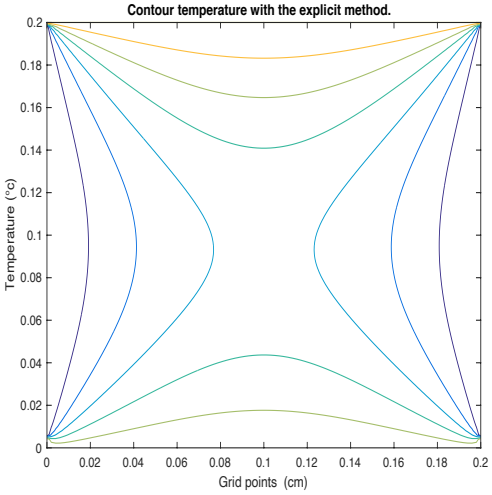


Figure 1. Contour of temperature with the explicit method with 15 × 15 points.

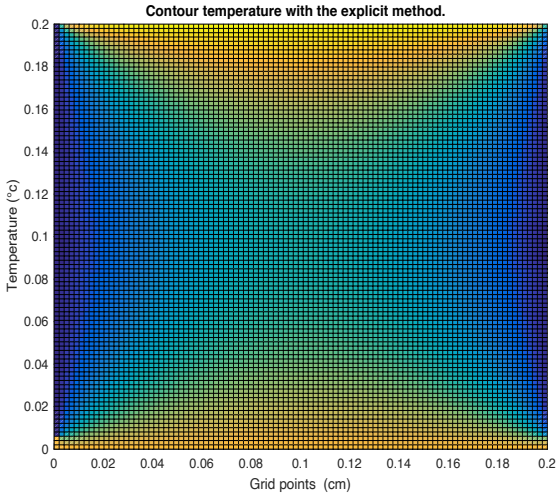


Figure 2. Contour of temperature in 2D.

In reference [12] a comparative study of the analytical and numerical methods to solve the Laplace equation is carried out and proved the equivalence of Equation (6) and Equation (11). A relevant result of this research shows that equivalence of experimental results and numerical behavior. The Figure 1 and Figure 2 proved this equivalence with the use of the temperature curve. The physical behavior of the studied phenomenon is fully understood by moreover the simplicity of the numerical method in contrast to the analytic solution presented in section 2.

This research reveals a deep connection between the experiment of section 2 and the mathematical model proposed in the Equation (11). This fact is reflected in the dynamics of heat conduction in the aluminum plate is determined in the Figure 2.

5. Conclusion

The experiment of heat conduction on the surface of the aluminum plate with the use of crystal liquid film as described in section 2, is modeled by the analytical Equation (6) of the Laplace equation, Equation (4). This research finds another mathematical model to describe the experiment in section 2, which has two clear advantages on the mathematical model of Equation (6). First of all, the numerical model is easy to get from the Laplace's Equation (4) and Equation (5) as verified in sub-section 3.2. Secondly, the computational implementation of the numerical scheme of Equation (12) shows the dynamics of the process of heating the aluminum plate with the use of crystal liquid film as can be verified with the Figure 1 and Figure 2.

References

[1] Villamizar Laguado W 2016 *Plan de Desarrollo para Norte de Santander* (Gobernación de Norte de Santander: San José de Cúcuta)

[2] Guerrero Gómez G, Espinel Blanco H 2013 comparación del consumo energética y propiedades finales de los productos cocidos entre hornos artesanales a cielo abierto y un horno Hoftman en las Ladrilleras del Municipio de Ocaña en Norte de Santander *XX Congreso Peruano de Ingeniería Mecánica, Eléctrica y Ramas Afines (CONIMERA)* (Peru: INICTEL)

[3] Guerrero Gómez G, Marrugo Carrazo D, Gómez Camperos J 2015 Desarrollo de instrumento virtual enfocado en la adquisición de datos para generar perfiles de temperatura en hornos *Revista Ingenio UFPSO* **8** 47
[4] Guerrero Gómez G, Espinel Blanco H, Sánchez Acevedo H 2017 Análisis de temperaturas durante la cocción de ladrillos macizos y sus propiedades finales *Revista Tecnura* **21(51)** 118
[5] Andrienko D 2018 Introduction to liquid crystals *Journal of Molecular Liquids physics* **267** 520
[6] Ondris R, Crawford G, and Doane J 1992 The phase of the future *Physics Teacher* **30** 332
[7] Zocchi G, Moses E, and Libchaber A 1990 Coherent structures in turbulent convection. An experimental study *Physica* **166** 387
[8] Canova B 1990 Laplace's equation in freshman physics *American Journal of Physics* **60** 135
[9] Bacon M 1994 Heat, light, and videotapes: Experiments in heat conduction using liquid crystal film *American Journal of Physics* **63** 359
[10] Crow T T 1987 Solutions to Laplace's equation using spreadsheets on a personal computer *American Journal of Physics* **55** 817
[11] Evans D, Abdullah A 1985 A new explicit method for the diffusion-convection equation *Computers and Mathematics with Applications* **11** 145
[12] Hart F 1989 Validating spreadsheet solutions to Laplace's equation *American Journal of Physics* **57** 1027