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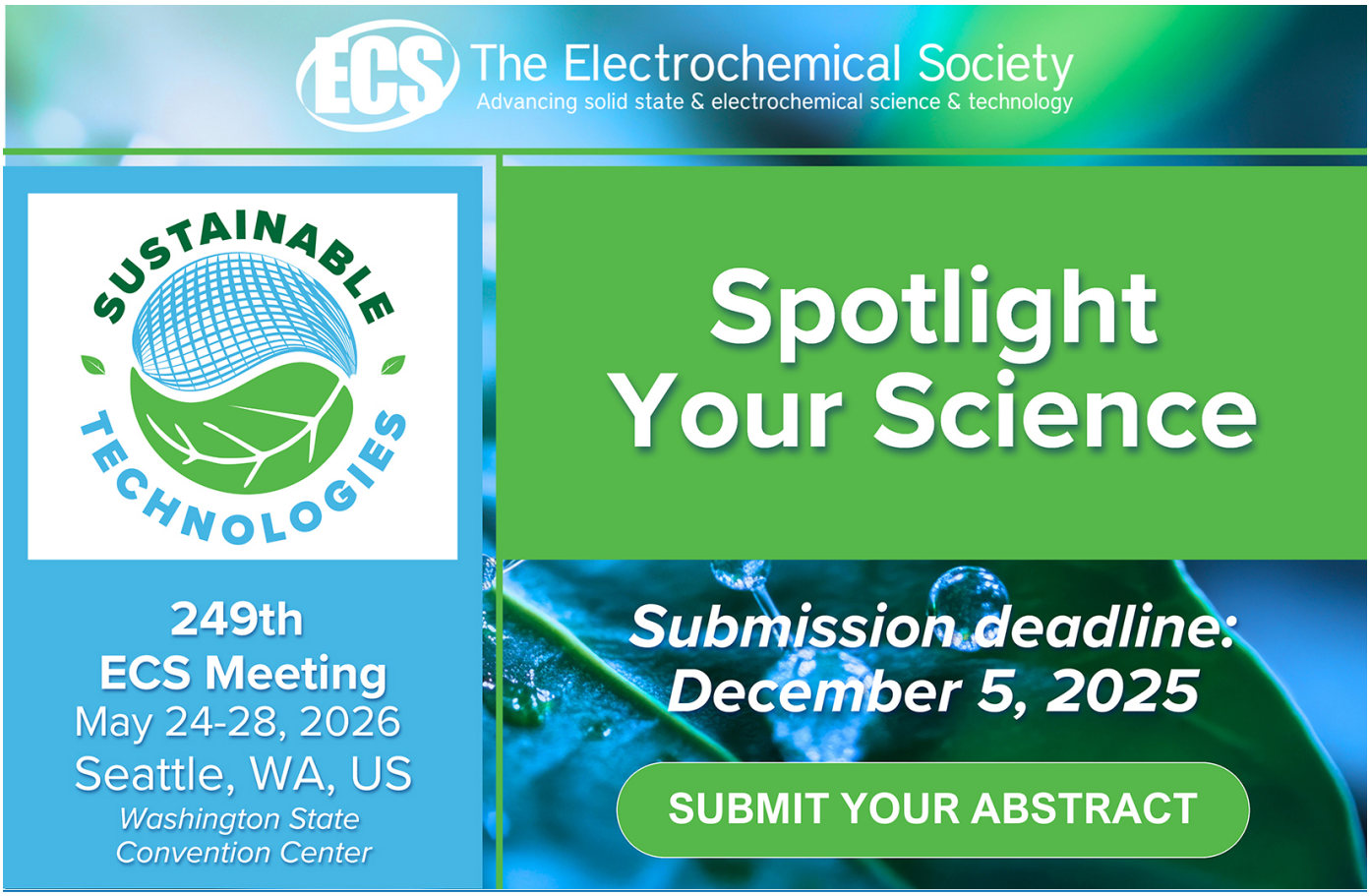
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To cite this article: N Afanador García *et al* 2020 *J. Phys.: Conf. Ser.* **1645** 012008

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Journal of Physics: Conference Series **1645** (2020) 012008 doi:10.1088/1742-6596/1645/1/012008

Evaluation of uncertainty in determining the physical properties of concrete using Bootstrap

N Afanador García[1], C J Noriega Sanchez[2], and C Nolasco Serna[3]

[1] Grupo de Investigación en Ingeniería Civil, Universidad Francisco de Paula Santander, Seccional Ocaña, Colombia

[2] Grupo de Investigación en Tecnología y Desarrollo en Ingeniería, Universidad Francisco de Paula Santander, Seccional Ocaña, Colombia

[3] Grupo de Investigación de la Facultad de Educación Arte y Humanidades, Universidad Francisco de Paula Santander, Seccional Ocaña, Colombia

E-mail: nafanadorg@ufpso.edu.co, cjnoriegas@ufpso.edu.co

Abstract. This investigation analyzed the uncertainty of compression resistance in the average statistics, standard error, asymmetry, kurtosis and confidence intervals taking into account the Bootstrap method. The implementation of the Bootstrap method in determining the physical properties of the material used in the construction of buildings, is a fundamental tool to establish the physical behavior of existing buildings. The application of the Bootstrap method allowed estimating the statistics for the reported dosages with different samples; 1:2:2, allowing to estimate statistics. The results allowed to determine the probability density function of the average value of the compressive strength and with it, probabilistic resistance values can be established. This research determined statistical results of the compressive strength vital to carry out revision, vulnerability and structural reinforcement studies of existing buildings in the municipality of Ocaña, Colombia.

1. Introduction

The compressive strength of concrete by destructive tests is determined using the standard method of compressive strength test of cylindrical concrete specimens [1] and that derived from the extraction of existing building cores. Compressive strength is an important parameter in existing buildings that sometimes cannot be obtained and can be determined from small samples present in material resistance laboratories in the city where the construction is located. This mechanical property of concrete has different uncertainty parameters that make its probabilistic analysis necessary.

Measurement processes are probabilistic in nature and measurement models are often complex [2, 3]. These characteristics; complexity and randomness make measurement data analysis an area of application for simulation methods. Among the methods applied in the analysis of measurement data, the Monte Carlo simulation method and the Bootstrap resampling method, are very useful for determining measurement statistics.

The analysis of data and its manipulation has been possible with the use of computers that allow the implementation of modern techniques of analysis before not possible. The use of computers has enriched the work of exploration and statistical analysis of data, such as simulation techniques and re-sampling methods. This symbiosis between computers and data

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analysis has allowed the use of data demonstration techniques, among which are Jackknife, randomization tests, cross-validation and Bootstrap [4]. The Bootstrap method has been used in different fields such as; paleomagnetic data analysis [5], psychology [6], geotechnics [7], forecasting of extreme events [8], economics [9], hydrology [10], among others.

In Ocaña, Colombia, structural reinforcements are being made to buildings built before 2010 [11] with few extractions of structural element cores. Also, building reviews are carried out with little or no information on the compressive strengths of concrete. This work aims to establish a method that allows establishing statistics of the compressive strength of concrete with small samples for use in structural design reviews and vulnerability studies and structural reinforcement of existing buildings.

2. Method

The Bootstrap method will be used to estimate the behavior of the average compressive strength of concrete cylinders for small samples, to estimate average, probability density function and confidence interval.

2.1. Bootstrap method

It was initially implemented in 1979 by [12], it is a computationally intensive method that is based on the re-sampling of the observed data, and it is described in more detail in [13–16]. Bootstrap methods are a type of non-parametric Monte Carlo methods that attempt to estimate the distribution of a population by re-sampling. The expression Bootstrap can be referred to Bootstrap non-parametric or Bootstrap parametric. The parametric Bootstrap is referred to samples taken known the probability density function ($f_{\boldsymbol{X}}$), meanwhile, in the case of non-parametric the $f_{\boldsymbol{X}}$ is not specified.

The distribution of the finite population represented by the sample can be considered as a pseudo population with characteristics similar to the true population. By repeatedly generating random samples of this pseudo population (resampling), the sample distribution of an arbitrary estimator can be estimated $\hat{\theta}\left(x_1, x_2, \cdots, x_n\right)$ where the cumulative density function of the sample $X_1, X_2, \cdots, X_n$ ($F_{\boldsymbol{n}}(x)$) is shown in Equation (1).

$$F_{\boldsymbol{n}}(x) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{I}_{\boldsymbol{X}_i < x}\left(x\right), \tag{1}$$

where $\mathbb{I}_{\{x \geq a\}}(x)$ is the function of the set indicator $\{x\, \epsilon\, \mathbb{R} : x \geq a\}$, see Equation (2).

$$\mathbb{I}_{\{x \geq a\}}(x) = \begin{cases} 1 & if\ x \geq a \\ 0 & any\ \ other\ \ case \end{cases}. \tag{2}$$

For Equation (1), instead of the theoretical cumulative density function (F). If an estimator $\theta(F) = \int h(x)dF(x)$ is desired, an obvious candidate is $\theta(F_n) = \int h(x)dF_n(x)$. When the X_i's are independent and identically distributed (iid) Glivenko-Cantelli's theorem [17] states that the sup-norm converges from F_n a F, and therefore guarantees that $\theta(F_n)$ is a consistent estimator of $\theta(F)$. It is common to denote a Bootstrap with a superscript "*", so a bootstrap sample can be indicated $\boldsymbol{X}^{*i} = (X_1^*, X_2^*, \cdots, X_n^*) \sim F_n$. Where the X_i^*'s are equal to one of the x_j's and that some value of x_j may appear several times in X^{*i}. Then for $X^{*i}, ..., X^{*B}$, $\theta(F_n)$ can be approximated by a Bootstrap estimator, see Equation (3).

$$\hat{\theta}(F_n) \cong \frac{1}{B}\sum_{i=1}^{B} h\left(x^{*i}\right). \tag{3}$$

As B increases the approximation is more precise. If θ a population parameter of interest and $\hat{\theta}$ an estimator of θ, Bootstrap estimate of distribution $\hat{\theta}$ is obtained from,

(i) For each Bootstrap replica, indexed by $b = 1, 2, ..., B$.
   (a) Generate the sample $x^{*(b)} = \{x_1^*, x_2^*, \cdots, x_n^*\}$ by sampling with replacement from the observed sample $\{x_1, x_2, \cdots, x_n\}$.
   (b) Calculate the b-th replica of $\hat{\theta}^{(b)}$ from the b-th bootstrap show.

(ii) The Bootstrap estimate of $F_{\hat{\theta}}(\cdot)$ is the empirical distribution of the replicas $\hat{\theta}^{(1)}$, $\hat{\theta}^{(2)}$, $\cdots$, $\hat{\theta}^{(B)}$.

The Bootstrap estimate of the standard error of an estimator $\hat{\theta}$ is the standard deviation sample of Bootstrap replicas $\hat{\theta}^{(1)}$, $\hat{\theta}^{(2)}$, $\cdots$, $\hat{\theta}^{(B)}$, see Equation (4).

$$\hat{\sigma}_{\hat{\theta}_b^*} = \sqrt{\frac{1}{B-1}\sum_{i=1}^{B}\left(\hat{\theta}_i^* - \hat{\mu}_{\hat{\theta}_i^*}\right)^2}, \tag{4}$$

where $\hat{\mu}_{\hat{\theta}_i^*}$ is shown in Equation (5).

$$\hat{\mu}_{\hat{\theta}_b^*} = \frac{1}{B}\sum_{i=1}^{B}\hat{\theta}_i^*. \tag{5}$$

According [12, 13], the number of replicas needed to obtain a good estimate of the standard error is not high; B = 50 is usually sufficient, however, if necessary, B much larger for the estimation of confidence intervals. If $\hat{\theta}$ is an arbitrary estimator of $\theta(F)$, the bias and the error distribution of $\hat{\theta}$ can be estimated using the Equation (6).

$$b_n = E_F = \left[\hat{\theta} - \theta(F)\right], \quad P_F\left(\hat{\theta} - \theta(F) \leq u\right). \tag{6}$$

They can be approximate replacing F_n by F. The Equation (7) illustrates the bootstrap estimator of bias.

$$b_n = \hat{\mu}_{\hat{\theta}_b^*} - \hat{\theta} \approx \sum_{i=1}^{B}\left\{\hat{\theta}(F_{ni}^*) - \hat{\theta}(F_n)\right\}. \tag{7}$$

A positive bias will indicate that $\hat{\theta}$ tends to overestimate the average of θ, and a negative bias will indicate that $\hat{\theta}$ underestimate the average. Meanwhile the confidence interval $\left[\hat{\theta} - \beta, \hat{\theta} - \alpha\right]$ in θ can be constructed by imposing the restriction, see Equation (8).

$$P_{F_n} = \left(\alpha \leq \hat{\theta}(F_n) - \theta(F_n) \leq \beta\right) = c. \tag{8}$$

In (α, β), where c is the desired level of confidence. There are several ways to obtain confidence intervals using Bootstrap; within which we can mention: (i) Standard normal bootstrap, (ii) Bootstrap percentile, (iii) Basic bootstrap, (iv) Bootstrap t, and (v) Bootstrap improved Mouth.

The standard normal Bootstrap confidence interval is the closest approximation more simple, although it is not necessarily the best, since it requires several assumptions: $\hat{\theta}$ it must have a normal distribution or be a half sample and the sample size is large enough $\hat{\theta}$ must be unbiased for θ. Bootstrap confidence interval percentile, are estimates of the barrels of the empirical distribution are estimates of the barrels of the sample distribution of $\hat{\theta}$, so that these random quartiles reproduce the true distribution better when the distribution of $\hat{\theta}$ is not normal. Efrén

and Tibshirani proved that the percentile interval has theoretical advantages over the standard normal range and better behavior in practice. The basic Bootstrap confidence interval transforms the distribution of Bootstrap replicas by subtracting the observed statistic. The barrels of the transformed sample are used to determine the confidence limits. The Bootstrap t confidence interval does not use the Student t distribution as a reference distribution, but generates by sampling the sample distribution of a "type t" statistic. If $x = (x_1, x_2, \cdots, x_n)$ is the random sample observed, the corresponding statistic observed $\hat{\theta}$. The Bootstrap t confidence interval at the trust level 100 $100(1-\alpha)\%$ is given in the Equation (9).

$$\left(\hat{\theta} - t^*_{1-\frac{\alpha}{2}} \cdot \hat{\sigma}_{\hat{\theta}}, \hat{\theta} - t^*_{\frac{\alpha}{2}} \cdot \hat{\sigma}_{\hat{\theta}}\right), \tag{9}$$

where $\hat{\sigma}_{\hat{\theta}}$, $t^*_{1-\frac{\alpha}{2}}$ and $t^*_{\frac{\alpha}{2}}$ is calculated as follow.

(i) Calculate $\hat{\theta} = \hat{\theta}(x)$.

(ii) For each Bootstrap replica, indexed by $b = 1, 2, \cdots, B$:
   (a) Generate the b-th Bootstrap sample $x^{(b)} = \left(x_1^b, x_2^b, \cdots, x_n^b\right)$ by sampling with replacement from x.
   (b) Calculate $\hat{\theta}^{(b)}$ from the b-th bootstrap show $x^{(b)}$.
   (c) Calculate or estimate the standard error $\hat{\sigma}_{\hat{\theta}^{(b)}}$ by re-sampling from $x^{(b)}$, no of x, independently for each Bootstrap sample.
   (d) Obtain the b-th replica of the statistician "t", $t^{(b)} = \frac{\hat{\theta}^{(b)} - \hat{\theta}}{\hat{\sigma}_{\hat{\theta}^{(b)}}}$.

(iii) Get the quartiles samples $t^*_{\frac{\alpha}{2}}$ and $t^*_{1-\frac{\alpha}{2}}$ of the ordered sample of replicas $t^{(b)}$, since the sample of replicas $t^{(1)}, \cdots, t^{(B)}$ is the reference distribution for "t".

(iv) Calculate $\hat{\sigma}_{\hat{\theta}}$ as the standard deviation shows the replicas $\hat{\theta}^{(b)}$.

(v) Determine confidence limits $\left(\hat{\theta} - t^*_{1-\frac{\alpha}{2}} \cdot \hat{\sigma}_{\hat{\theta}}, \hat{\theta} - t^*_{\frac{\alpha}{2}} \cdot \hat{\sigma}_{\hat{\theta}}\right)$.

Bootstrap confidence intervals improved are a variant of the percentile intervals that have better theoretical properties and provide superior performance in practice. Bootstrap t confidence interval has second-order accuracy and was the interval of confidence used in the analysis of the compressive strength of concrete (f'_c). A schematic representation of the Bootstrap procedure for estimating different statistics is indicated in Figure 1.

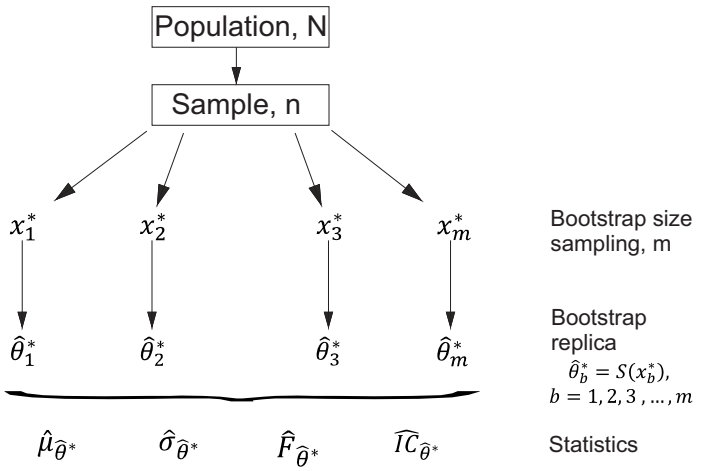


Figure 1. Schematic representation of the Bootstrap procedure.

In Figure 1 the Bootstrap procedure is illustrated, where from an observed sample it is possible by re-sampling with replacement to estimate different statistics of the variable f'_c. The compressive strength test of concrete specimens is carried out following the guidelines established in [1] for both 28-day cylinders and those extracted from existing buildings.

3. Results

Of the 56 results found between 2008 and 2018 in the "Laboratorio de Materiales y Resistencia Sísmica, Universidad Francisco de Paula Santander, Seccional Ocaña, Colombia" of the f'_c for a 1:2:2 dosage, it was possible to apply the Bootstrap methodology to estimate the average value of the f'_c, $(\mu_{f'_c})$, that is to say $\hat{\mu}^*_{f'_c}$. Histograms for different resampling are shown in Figure 2, with: 30, 100, 300, 1000, 3000 and 10000 with replacement.

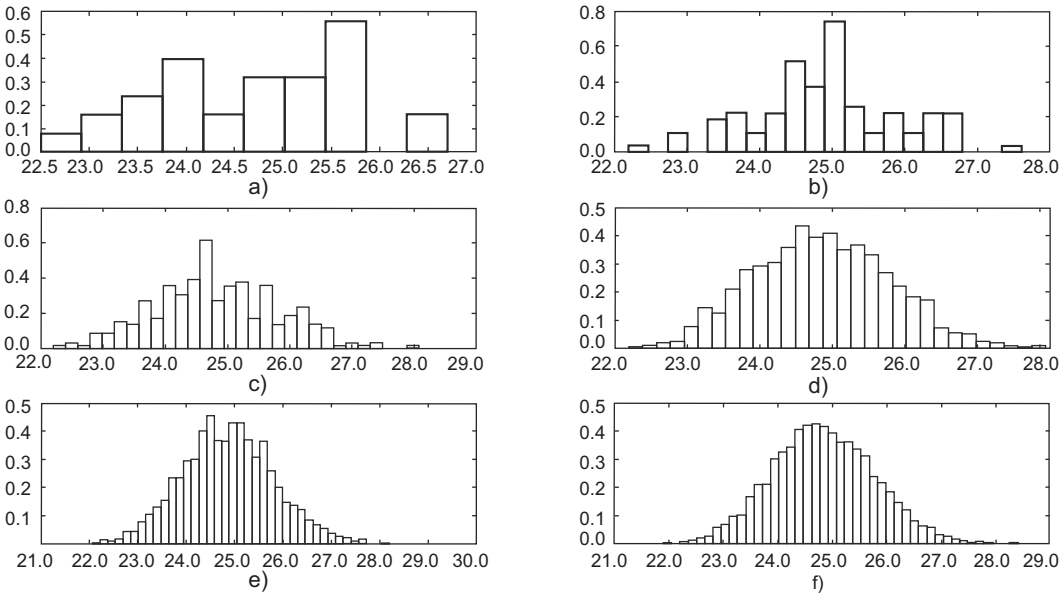


Figure 2. Bootstrap histogram for compressive strength with 1:2:2 dosing and resampling with replacement, (a) 30, (b) 100, (c) 300, (d) 1000, (e) 3000, and (f) 10000.

For small resampling (30), it is difficult to establish the physical behavior of the compressive capacity of the concrete, Figure 2(a). The same behavior is observed for a resampling $\leq$100, Figure 2(b). For resampling = 300, it is not possible to define the behavior of the ends of the histogram and nor the measures of central tendency, Figure 2(c), as for resampling equal to 1000, the mean and average values of the compressive strength show some trend, Figure 2(d). Although for resampling = 3000, the behavior of the ends of the histogram is adequate, the same does not occur for the center of the histogram, Figure 2(e). The average distribution of the $\hat{\mu}^*_{f'_c}$ presents a more regular distribution of the results as the resampling increases (10000), Figure 2(f). Figure 3 shows how the value varies $\hat{\mu}^*_{f'_c}$ regarding the assumed Bootstrap.

An analysis of the convergence of $\hat{\mu}^*_{f'_c}$ presented in Figure 3 indicates that by increasing the re-sampling number to 10000 the value of the $\hat{\mu}^*_{f'_c}$ converges to 24.84 MPa. The probability density functions for each resampling are indicated in Figure 4.

La probability density function that best defines the behavior of the $\hat{\mu}^*_{f'_c}$ it is the Bootstrap with 10000 resampling with replacement whose $f_{\boldsymbol{X}}(x)$ is a normal log and is illustrated in Figure 4 with μ = 24.84 MPa and standard deviation σ = 0.91 MPa. The $f_{\boldsymbol{X}}(x)$ that best fits the data was determined using the maximum likelihood estimator. The results of the 3rd

statistical moment for 10000 resampling with replacement indicated that the average value of bias or asymmetry is 1.08 and a standard deviation of 0.24 with a confidence interval of [0.60, 1.58] with a probability of 97.50%. For the 4th statistical moment for 10000 resampling with replacement, he indicated that the average value of the kurtosis is 3.54 with a standard deviation of 0.87 and a confidence interval of [1.80, 5.29] with a probability of 97.50%.

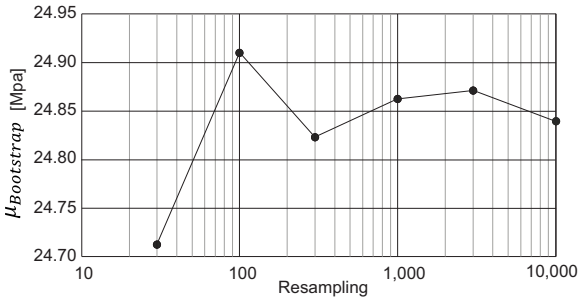


Figure 3. The convergence of average compressive strength with 1:2:2 dosage and different Bootstrap.

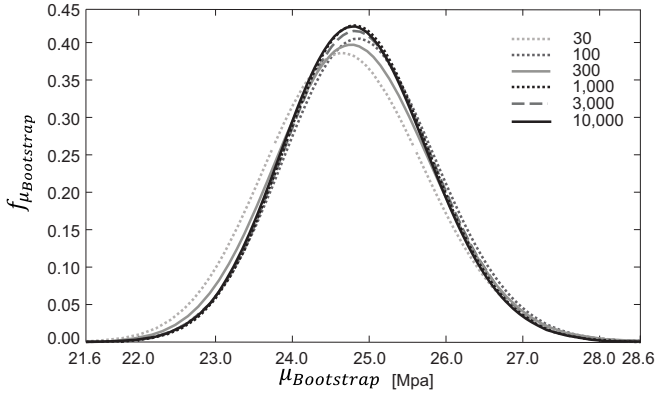


Figure 4. Probability density function of the average compressive strength with 1:2:2 dosage and different Bootstrap.

Product of the extraction of cores made to different structural elements of existing buildings by the "Laboratorio de Materiales y Resistencia Sísmica, Universidad Francisco de Paula Santander, Seccional Ocaña, Colombia" was reported 19 compressive strength of concrete cylinders. Applying the Bootstrap methodology with samples by resampling of 30, 100, 300, 1000, 3000, 10000 allowed to determine the probability density function for the different resampling and indicated in Figure 5. The probability density function that defines in Appropriate form the behavior of the average compressive strength of the core extraction cylinders was the Bootstrap with 10000 resampling replacements whose $f_{\boldsymbol{X}}(x)$ is a lognormal with $\mu = 16.42$ MPa, $\sigma = 1.61$ MPa. For the 3rd statistical moment with 10000 resampling with replacement, I found that the average bias value is -0.35 with a standard deviation of 0.39 and a confidence interval of [-1.17, 0.48] with a probability of 97.50%. For the 4th statistical moment for 10000 resampling with replacement, he indicated that the average value of the kurtosis is 1.90 with a standard deviation of 0.52 and a confidence interval of [0.82, 2.99] with a probability of 97.50%.

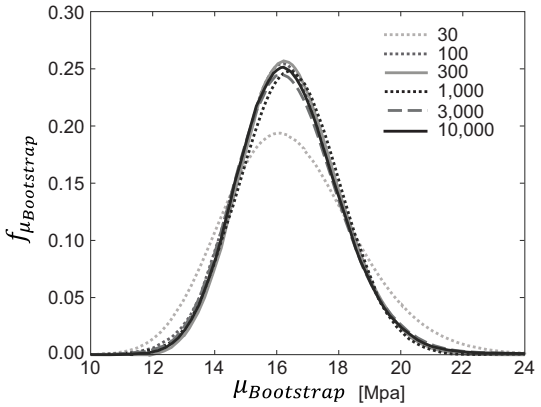


Figure 5. The Lognormal probability density function of the average compressive strength for core extraction with different Bootstrap.

4. Conclusions

This research work was able to determine the average value of the compressive strength of hardened concrete cylinders at 28 days with a 1:2:2 dosage for the purpose of reviewing structural designs in the absence of specific building results, and whose value is 24.84 MPa and the $f_{\boldsymbol{X}}(x)$ for the average values it is a lognormal, and this will allow estimating probabilistic values of compressive strength. The use of the Bootstrap technique in the experimental field with a reduced number of samples, allows determining the physical properties of the material, as well as estimating the physical behavior of structural elements.

References

[1] American Society of Testing Materials (ASTM) 2018 *Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens, ASTM C39/C39M* (West Conshohocken: American Society of Testing Materials)
[2] Bich W 1997 *The ISO Guide to the Expression of Uncertainty in Measurement: A Bridge Between Statistics and Metrology* (United State of America: Series on Advances in Mathematics for Applied Sciences)
[3] Afanador-García N 2016 A polynomial chaos expansion approach to the analysis of uncertainty in viscoelastic structural elements *DYNA* **83(199)** 172-182
[4] Efron B 1979 Bootstrap methods: another look at the jackknife *The Annals of Statistics* **7(1)** 1-26
[5] Tauxe L, Kylstra N, Constable C 1991 Bootstrap statistics for paleomagnetic data *J. of Geophysical Research: Solid Earth* **96(B7)** 11723-11740
[6] Ledesma R 2008 Introduccción al bootstrap. Desarrollo de un ejemplo acompañado de software de aplicación *Tutorials in Quantitative Methods for Psychology* **4(2)** 51-60
[7] Wei H W, Xie J, Chau H W, Si B C 2015 Evaluation of parameter uncertainties in nonlinear regression using Microsoft Excel Spreadsheet *Environmental Systems Research* **4(1)** 1-12
[8] Gomes M I, Oliveira O 2001 The bootstrap methodology in statistics of extremes—choice of the optimal sample fraction *Extremes* **4(4)** 331-358
[9] Mackinnon J G 2006 Bootstrap methods in econometrics *Economic Record* **82(1)** 2-18
[10] Lall U, Sharma A 1996 A nearest neighbor bootstrap for resampling hydrologic time series *Water Resources Research* **32(3)** 679-693
[11] Asociación Colombiana de Ingeniería Sísmica (AIS) 2010 *Norma Sismo Resistente Colombiana, NSR-10* (Colombia: Asociación Colombiana de Ingeniería Sísmica)
[12] Bradley E 1979 Computers and the theory of statistics: Thinking the unthinkable *SIAM Review* **21(4)** 460-480
[13] E Bradley, Tibshirani R J 1982 *An Introduction to the Bootstrap* (United State of America: CRC press)
[14] Hall P 2013 *The Bootstrap and Edgeworth Expansion* (New York: Springer-Verlag)
[15] Hjorth J S 2017 *Computer Intensive Statistical Methods: Validation, Model Selection, and Bootstrap* (United State of America: Routledge)
[16] Dimitris P 1995 *A Primer on Bootstrap Methods in Statistics* (United State of America: Purdue University)
[17] Billingsley P 1995 *Probability and Measure* (United State of America: John Wiley & Sons)