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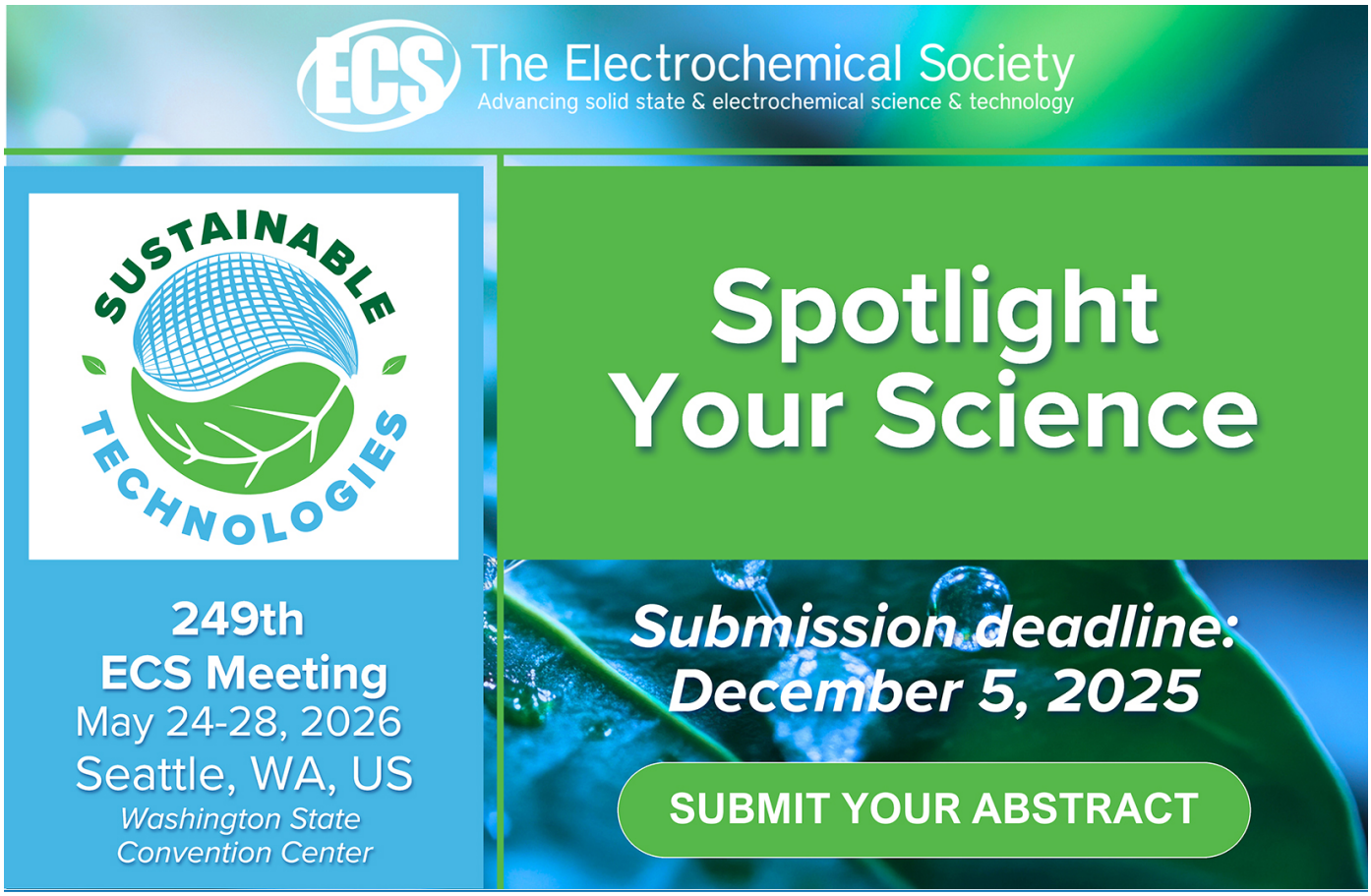
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Study of the modified Weibull function to analyze reliability in engineering components that fail and are repairable

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Abstrat. An engineering component is physically inoperative when it fails or has conditions that prevent it from operating and has a functional failure. The useful or operating times and the non-operating or downtimes are collected to determine the reliability or the probability of operation of the component during a period. The use of distributions is the most accepted method for determining component reliability. Among the main distributions used are the exponential, Log Normal, Weibull, etc. The Weibull distribution is widely used to estimate the reliability of a component, understood as the physical capacity of the element to function according to operating parameters. A study of three models based on Weibull modifications was conducted to determine the reliability of repairable components subject to maintenance activities. Two models represent the modified Weibull function with two parameters and an additional model that represents the Weibull function transmuted with failure rate function that fits the Davies curve. The characteristics, the physical interpretation, the applications, and the feasibility of using each model in analysis of reliability in elements and repairable in engineering are presented.

1. Introduccion

The current concept of industrial maintenance implies considering the equipment as assets that represent an investment for the company and for this reason they must be kept in operating conditions, that is, the management of physical assets implies the need to develop and implement a systematic way to program, execute and control maintenance activities according to an initial diagnosis of the equipment, with defined performance goals and a monitoring plan, oriented to the management of indicators to constantly evaluate the fulfillment of the proposed objectives [1]. Each organization establishes the indicators that it will manage according to its own reality and the sector of the economy in which it operates [2]. For the specific case of the maintenance department, various indicators can be defined such as: labor, turnover or levels of spare parts and inventory, quality indicators, maintenance not executed, mean time between failures (MTBF), mean repair time (MTTR), reliability, maintainability, and availability, among many others.

Due to its importance in determining the behavior of the equipment according to its operational context, as world-class maintenance indicators are reliability, availability, and maintainability (RAM), which manage the activities required with maintenance in any type of organization regardless of the sector of the economy in which it is developed [3]. To manage maintenance under the RAM analysis there are various applicable methodologies. These vary from the use of equipment performance measurements based on useful times and maintenance downtime, among others. In addition, they

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allow performing point calculations from averages to complex statistical and probabilistic models. The most appropriate methodology for each organization depends, among other aspects, on the degree of scientific and technical maturity, the characteristics of the environment and the possibility of repair or not of each equipment [4]. Parameterized distribution fitting methods are widely accepted in all types of industry for both repairable and non-repairable items. The maintenance strategy becomes more complicated as the system becomes more complex and requires greater economic and availability requirements, which must be carried out based on analyzes supported by data related to the state of the equipment [5]. RAM studies aimed at guaranteeing the operability of the equipment, use probability distributions such as models: normal, exponential, Log Normal, Weibull, Gamma, Poisson, Hasting and Rayleigh [6]. Due to its fit to the Davies curve that represents the probability of failure of a component or element in relation to the service time, the distribution most widely used by various researchers in this type of analysis is the Weibull [7]. An analysis of three proposed probabilistic models is carried out to analyze RAM using mathematical modifications to the Weibull function. The parameters required to be applied are identified, their physical implications and possible benefits at the time of being used to carry out operational reliability analysis in repairable systems, components, and engineering elements.

2. Metodologia

Two models representing the modified Weibull function with two parameters were studied, a transmuted Weibull model with a failure rate function that fits the Davies curve. The work was developed in four stages as shown in Figure 1. Initially, a description of each model was made with its characteristics and the way to estimate the parameters of each function. The three models were validated with a series of 33 data corresponding to useful times and non-operative times of a device reported in the literature by [4].

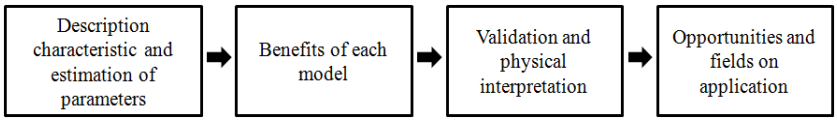


Figure 1. Methodology applied by stages.

2.1. Description, characteristics, and estimation of parameters

Additive Weibull model (AW) for reliability analysis was presented by [8], combines two Weibull distributions, one with a decreasing failure rate and the other with an increasing failure rate, uses four parameters. In his work [9] he presents some simplifications of the model and discusses the parameter estimation methods based on the graphical estimation technique. The failure rate probability density function is given by Equation (1).

$$f(t) = at^b + ct^d, \tag{1}$$

where $t, a, c \geq 0$, $b > 1$ and $d < 1$, its parameters of scale and shape. Table 1 shows the reliability function, the non-reliability function, the failure rate, and the mean time between failures.

Table 1. Description and characteristics of the model AW.

Reliability function	Unreliability function	Failurerate function	MTBF
$R(t) = e^{-at^b - ct^d}$	$F(t) = 1 - e^{-at^b - ct^d}$	$\lambda(t) = ab(at)^{b-1} + cd(ct)^{d-1}$	$MTBF = \int_0^\infty e^{-(at)^b - (ct)^d} dt$
Para $t \geq 0$	Para $t \geq 0$	Para $t \geq 0$	

Parameter estimation is performed graphically, through least squares linear regression; Equation (2) and Equation (3) allow us to calculate the values of “x” and “y”, respectively.

$$x = \ln t, \quad (2) \qquad y = \ln\left[\ln\left(\frac{1}{R(t)}\right)\right]. \quad (3)$$

When graphing and doing the linear regression, the equation of the line is given by Equation (4).

$$y = \beta \ln t + \ln\left(\frac{1}{\alpha}\right). \quad (4)$$

For small times you can approximate Equation (5) and for long times you work with Equation (6).

$$y = d\ln t + \ln c, \quad (5) \qquad y = b\ln t + \ln a. \quad (6)$$

The new additive weibull model (NAW) proposed by [10] in his work consists of a modification to the additive Weibull model (AW) proposedby [8], looking to adapt the failure rate function to the shape of the bathtub curve. This new modification to the traditional Weibull model allows better adjustments of the failure rates to the curve of the bathtub and can be used to model the reliability of systems that are at the beginning of the useful life or with a considerable time of service It is even applied to model wear on mechanical components and in some cases electrical ones. For wear and fatigue cases, this model behaves better than when using the exponential distribution, since it makes a good fit to the data. The lifetime distribution for NAW arises from analyzing the integrated beta distribution with appropriate limits [11], the reliability function is given by the parametersa > 0, $b \geq 0$ and $\eta > 0$, where η is scale parameter [10].

Table 2. Description and characteristics of the model NAW.

Reliability function	Unreliability function	Failurerate function
$R(t) = e^{-at^b e^{\eta t}}$	$f(t) = a(b+\eta t)t^{b-1}e^{\eta t}e^{-at^b e^{\eta t}}$	$\lambda(t) = a(b+\eta t)t^{b-1}e^{\eta t}$

The failure rate function$\lambda(t)$, depends only on the form factor b, y t^{b-1}, because the other two parameters have no influence.

Case 1. When $b \geq 1$, the failure rate shows that:

- $\lambda(t)$ increases with t,which implies an increase in the failure rate function.
- $\lambda(0) = 0$ if $b > 1$; $\lambda(0) = ab$ if $b = 1$.

Case 2. When $0 < b < 1$

- $\lambda(t)$ initially decreases and then increases which resembles the curve of the bathtub.
- $\lambda(t) \to 0$ when $t \to 0, \lambda(t) \to \infty$ when $(t) \to \infty$.
- The derivative of $\lambda(t)$ intersects the $t-$ axis only once at t^x for t 0, $\lambda(t)$ decreases for $t < t^x$ and is increasing for $t > t^x$ which is given by $t^x = \frac{\sqrt{b}-b}{\eta}$.

The interesting characteristic is that t^x decreases as η increases, which is interpreted in the following way, the failure rate function for the classical weibull distribution can be constant, decreasing or increasing and for the new model it differs because due to the additional factor e^ηt which can be seen as an increasing acceleration factor for both t^x and η. The estimation of parameters for this model can be done by graphical method with linear regression by least squares or by the method of maximum likelihood. In the NAW model the values of "x" and "y" are determined by Equation (7) and Equation (8) respectively.

$$x = \ln t, \quad (7) \qquad y = \ln a. \quad (8)$$

From linear regression, the straight line is given by Equation (9).

$$y = b\ln t + \eta e^{x}. \tag{9}$$

The modified transmuted Weibull model (TMW), like previous modifications to the traditional Weibull distribution, seeks to become an option to perform failure and reliability analysis in repairable systems [12]. Because it is a distribution that arises from the modified Weibull model, it is quite flexible, it uses four parameters, therefore, it can be used to model reliability of various failure modes. From the analysis of the instantaneous failure rate function, it is observed that the failure rate pattern can increase and decrease according to the data based on the lifetime of the component. It uses four parameters, two of form β and η, one of scale α and the transmuted γ, for the range of values α, β, $\eta > 0$, and $-1 \leq \gamma \leq 1$, the failure probability density function is given by the Equation (10).

$$f(t) = \left(\alpha + \beta\eta t^{\beta-1}\right)e^{\left(-\alpha t-\eta t^{\beta}\right)\left(1-\gamma+2\gamma e^{\left(-\alpha t-\eta t^{\beta}\right)}\right)}, \tag{10}$$

where the shape parameters represent different patterns of the transmuted model and are positive, the scale parameter represents the characteristic life and is also positive.

Table 3. Description and characteristics of the model TMW.

Reliability function	Unreliability function	Failure rate function
$R(t) = 1-\left(1-e^{\left(-\alpha t-\eta t^{\beta}\right)}\right)\left(1+\gamma e^{\left(-\alpha t-\eta t^{\beta}\right)}\right)$	$F(t) = \left(1-e^{\left(-\alpha t-\eta t^{\beta}\right)}\right)\left(1+\gamma e^{\left(-\alpha t-\eta t^{\beta}\right)}\right)$	$\lambda(t) = \frac{\left(\alpha+\beta\eta t^{\beta-1}\right)e^{\left(-\alpha t-\eta t^{\beta}\right)\left(1-\gamma+2\gamma e^{\left(-\alpha t-\eta t^{\beta}\right)}\right)}}{1-\left(1-e^{\left(-\alpha t-\eta t^{\beta}\right)}\right)\left(1+\gamma e^{\left(-\alpha t-\eta t^{\beta}\right)}\right)}$

For the random sample of times of size n, the sample is ordered by Equation (11).

$$E\left(F(T_i)\right) = \frac{i}{n+1}. \tag{11}$$

The least squares estimators are obtained by minimizing by means of Equation (12).

$$Q(\alpha,\beta,\eta,\gamma) = \sum_{i=0}^{n}\left(F(T_i) - \frac{i}{n+1}\right)^2. \tag{12}$$

To determine the parameters by the maximum likelihood method, Equation (13) is used.

$$L\left(t_1,\ldots t_{2,}\alpha,\beta,\eta,\gamma\right) = \prod\left(\alpha + \beta\eta t^{\beta-1}\right)e^{\left(-\alpha t-\eta t^{\beta}\right)\left(1-\gamma+2\gamma e^{\left(-\alpha t-\eta t^{\beta}\right)}\right)}. \tag{13}$$

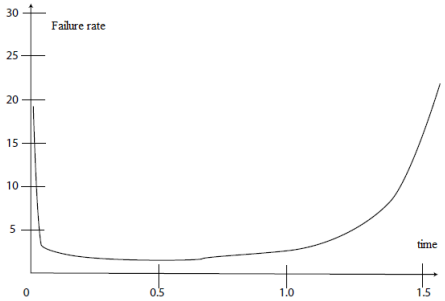
The modified Weibull extension with bathtub-shaped failure rate function proposed by [13].

2.2. Physical interpretation

The physical background of the AW model is clear: a component fails due to the occurrence of damage and there are different failure modes associated with the same component. Each failure mode affects the component differently. Component failure modes indicate the associated failure pattern, and this is the component's life stage according to the three phases of the Davies curve, for any type of component studied as mechanical, electrical, or electronic. When the failure rate graph reflects values of $\beta < 1$, the failure pattern corresponds to the assembly, targeting or synchroinization stage known as infant mortality, on the other hand, when the failure rate reflects values of $\beta = 1$, failures do not they are related to the operating time, finally, if the failure rate reflects values of $\beta > 1$, the component

increases the probability of presenting damage or it may be at the end of its life cycle. Now, if it is considered that a component is affected by two main failure modes, each of them corresponds to a Weibull distribution with a different parameter and when graphing the failure rate function with different parameters, it is observed that the model is It adapts to the three phases of the bathtub curve and is applicable for data series that represent the useful and non-operating times of the component, without knowing if the type of failure that occurs is due to electrical, mechanical or electronic failures.

Figure 2 and Figure 3 show the failure rate function for different values of the parameters a, b, c and d graphed for the 33-time records reported by [4] according to the models presented by [8] and [10].



Figuere 2. Failure rate AW a=1, b=5, c=2, d=5.

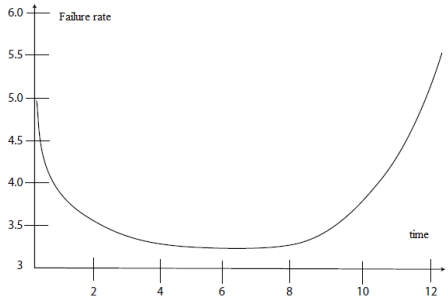


Figure 3. Failure rate AW a=1, b=5, c=5, d=9.

The NAW model is related to the traditional Weibull distribution when $\eta = 0$ and to the Rayleigh distribution when $\Theta = 0$. For a series of data that relate to the failures of a component or systems with which reliability analysis is required, this model makes it possible to quickly make the decision regarding the viability of the distribution according to the characteristics of the data in the series. Through the graphical approach, visual evaluation is made as the basis for deciding whether the failure rate function of the data sample has a shape like a typical bathtub curve. Figure 4 shows the graph of the failure rate function of NAW and the Figure 5 of TMW models respectively with different estimated parameter values for the 33-time records reported by [4].

The physical interpretation of the TMW model indicates based on the failure rate function that: when $\beta = 5$ the distribution tends to be strictly increasing; when $\beta<1$, the failure rate is decreasing and represents infant mortality or premature failures of the break-in period; when $\beta = 1$, the failure rate is constant and represents the failures during the useful life or period of operation; when $\beta > 1$, the failure rate function is increasing and represents damage that occurs after periods of useful life. This period is better known as the wear period. This distribution resembles Rayleigh for $\beta = 2$ and for $\beta = 1$ it coincides with the exponential distribution. Therefore, the WTM distribution is very flexible for reliability analysis.

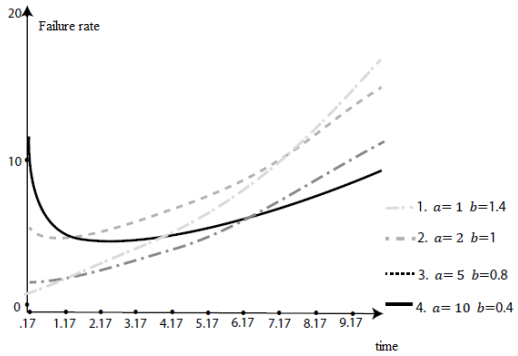


Figure 4. Failure rate NAW different parameters.

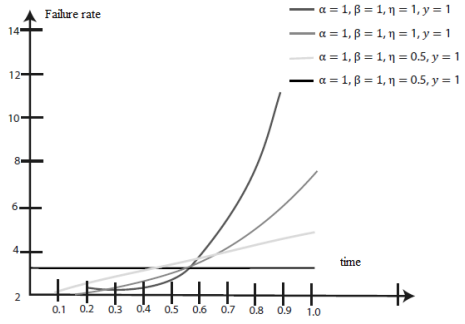


Figure 5. Failure rate TMW different parameters.

2.3. Opportunities and fields of application

The AW model is one of the distributions that best fits the curve of the bathtub, therefore it is suitable for describing the lifetime of any mechanical, electrical, or electronic component. Studies and research that propose methodologies for the estimation of its parameters should be deepened, with the precept of using the random series with all the data of the component's lifetime and not separating them in the running-in, maturity and aging phases to carry out the test's analysis. That is, future studies should focus on the estimation of parameters that improve the fit of the model.

The NAW model is useful for modeling data series that relate to the life of a component or system at the level in theoretical studies with some partial results in practice. The difficulty in using the model is mainly the correspondence between the series and the results obtained, since goodness adjustment must always be made with the extreme values for the model to be applicable in practice. In general terms, the best results are obtained in the reliability study of components that are put into operation and quickly reach an operating performance, that is, the break-in period is very short, and faults are easily solvable. The TMW model is useful for making decisions regarding the cyclical change or replacement of components in engineering based on reliability and failure rate. Likewise, its use is interesting to improve the reliability of products that are introduced to the market and whose infant mortality rate is very high due to design problems, manufacturing, or installation defects.

3. Resultados

The AW model can be reduced to just two parameters for ease of application. When considering a special case when $a = c$ and d = 1 / b, when $b > 1$ and therefore $d = 1 / b < 1$, the model is reduced to two parameters and the curve shape of the bathtub is preserved in the failure rate function. The AW model allows determining the optimal moment for the change of components, in addition, it is useful for the analysis of reliability of products, systems and in the estimation of hours of replacement of parts and time of service. Use the failure rate function to determine the replacement period before the risk of failure increases. In other words, future studies should focus on the estimation of parameters that improve the fit of the model. From the application of the NAW model as described, numerous studies and theoretical applications can be carried out with the use of real data series with the purpose of demonstrating and teaching the distribution during maintenance learning. It offers great expectations as the parameters that are estimated for the entire data series are adjusted to the failure rate function in the form of a bathtub. Of the models that were found, it is one of the most promising and it is necessary to deepen studies and practical applications. It is possible to experiment with the model through studies that allow to conclude its advantages and the degree of adjustment to the curve of the bathtub when the entire data series is analyzed, and the small times (corresponding to the break-in period) are not separated from the times long (wear or aging).

The WTM distribution offers the possibility of studying the relationships that exist between the shape parameter with other functions such as unreliability, reliability, instantaneous failure rate and cumulative failure function. At the research level, it is interesting to compare the data obtained by using the WTM model with those of other traditional and non-traditional parametric and non-parametric distributions, to define guidelines that are necessary for its application. It is necessary to deepen the estimation of parameters to adjust the results to the shooting or synchronization phase of the equipment. This work shows that the theoretically proposed models have practical applications according to the validation carried out based on the operating and non-operating times of a device reported by [4]. It is shown that when graphing the failure rate functions for each model studied using scale parameters different it is possible to identify the reliability of a component in relation to the parameter of form β that indicates the physical performance of the equipment and its performance under normal operating conditions according to the phases of the bathub curve. According to the results, the AW model can be used in the reliability analysis of mechanical, electrical or electronic components at any stage of their life cycle, the NAW model is recommended for analysis in mechanical equipment that is at the end of its life cycle and is required to determine the appropriate time to replace them, and the WTM model for the same analysis at the end of the useful life of the

mechanical components and additionally for electrical and electronic equipment with random physical failures that do not depend on their life cycle.

4. Conclusions

The applications of the models allow to graph the function of the failure rate of a component in relation to the measurement of the useful and non-operative times, without considering the origin of the failure, for which there are other methods such as failure analysis. This type of analysis RAM adapts the value of parameter B to the three phases of the bathtub curve to determine the physical state of the equipment in relation to operating and non-operating times, as an indicator of its probability of failure to consider the actions of maintenance appropriate to each case. The AW model offers great expectations as the parameters that are estimated for the entire data series are adjusted to the failure rate function in the form of a bathtub. Of the models that were found, it is one of the most promising and it is necessary to deepen studies and practical applications.

Although the NAW is a distribution that was developed mainly to perform analysis at a theoretical level, due to its flexibility and ability to process data series by the graphical method quite simply, good results are expected from its application. For this, it is necessary to deepen in investigations that are conclusive and support the use of this distribution in studies that relate to the periods or frequency required for the change of elements, to program their own preventive maintenance actions. The characteristics of the WTM distribution provide the possibility of studying the relationships that exist between the shape parameter with other functions such as unreliability, reliability, instantaneous failure rate and cumulative failure function. On a practical level, expectations with the use of the model are focused on the analysis of components subjected to fatigue and wear. Its results are comparable with those of traditional distributions and with modified models of these.

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