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Applications of thermodynamics to study the physical phenomenon of heat conduction

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Abstract. Thermodynamics can be understood as the discipline of the physical sciences, that studies theoretically and practically the different manifestations of energy. The study of thermodynamics is important in relation to the understanding of thermal systems in industry, as well as, supporting energetic processes in living organisms. In relation to the study of energy processes in the context of heat transfer, concepts from thermodynamics are relevant. In the present investigation, the process of heat conduction in a metal bar is analyzed by applying the heat equation and the concept of entropy variation. The first part of the research proposes a numerical method to solve the heat equation in addition to a set of finite difference equations describing the energetic behavior of the system. The numerical solution of the heat equation and the thermodynamic behavior of the system are studied by programming to demonstrate the fit of the results with the theoretical models. Finally, applications of the achieved results in engineering contexts are discussed.

1. Introduction

The field of physics responsible for the measurement of energetic processes is thermodynamics. The study of thermodynamics is important in relation to the understanding of thermal systems in industry, as well as, supporting energetic processes in living organisms [1]. The development of quantitative and qualitative qualities of heat conduction from the perspective of thermodynamic properties has been widely developed [2]. The analysis of energy effects derived from heat conduction in most applications uses mathematical models derived from the classical thermodynamic theory [3,4] by contrast numerical techniques for research on energy are important in recent years [5].

This research looks into the application of numerical methods to study the phenomenon of heat conduction in a metallic bar in a transient state to prove the stability of the system through the use of the entropy equations [6]. The benefit of the method proposed in the research will prove to be useful in the case of heat conduction in bricks. The research first of all proposes the method of solution of the heat equation and the derivation of the energy analysis equations. Secondly, using a computer program, the results are analyzed in comparison with the theoretical data to prove the accuracy of the proposed approach.

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2. Mathematical modeling and entropy conservation

The description of modeling the energetic process in heat conduction is described firstly through the approach and solution of the heat equation and finally the application of numerical methods to generate the energy conservation equations.

2.1. Formulation of the mathematical model

The law of conservation of energy [7] describing the behavior of heat flow in regions of space is represented by the Equation (1), where c is specific heat, ρ is density and Ψ is temperature.

$$\frac{d}{dt}\int_R c\rho\Psi dV = -\int_R \text{div}\,\phi dV + \int_R f dV. \tag{1}$$

Applying Fourier's law and the principle of continuity [7] to complete Equation (1) produces the heat Equation (2).

$$\frac{\partial\Psi}{\partial t} = \lambda^2 \frac{\partial^2\Psi}{\partial x^2}. \tag{2}$$

The function $\Psi(x,t)$ represents the temperature in the bar, where x is the position and t the time. The constant λ represents the thermal diffusivity of the bar. Taylor's theorem [8] applied to the temperature function guarantees the Equation (3) and Equation (4).

$$\Psi(x+h) = \Psi(x) + h\Psi'(x) + \frac{1}{2}h^2\Psi''(x) + \cdots, \tag{3}$$

$$\Psi(x-h) = \Psi(x) - h\Psi'(x) + \frac{1}{2}h^2\Psi''(x) + \cdots. \tag{4}$$

Addition of complete Equation (3) and Equation (4) gives the Equation (5).

$$\Psi(x+h) + \Psi(x-h) = 2\Psi(x) + h^2\Psi''(x). \tag{5}$$

From Equation (5), the representation of Ψ'' is defined in the Equation (6).

$$\Psi''(x) = 1/h^2\ [\Psi(x+h) - 2\Psi(x) + \Psi(x-h)]. \tag{6}$$

Subtraction of complete Equation (4) from complete Equation (3) gives Equation (7).

$$\Psi' = \frac{1}{2h}[\Psi(x+h) - \Psi(x-h)]. \tag{7}$$

Suppose that the metal bar of length L is divided into N equal parts of size $\Delta x = \frac{L}{N}$, substituting the complete Equation (6) and Equation (7) into the complete Equation (2) generates the discrete version of the heat equation formulated in Equation (8).

$$\frac{\Psi_i^{n+1} - \Psi_i^n}{\Delta t} = \lambda \frac{\Psi_{i-1}^n - 2\Psi_i^n + \Psi_{i+1}^n}{(\Delta x)^2}. \tag{8}$$

It is possible to prove that the solution of the heat Equation (8) is equivalent to solve a system of equations of N equations with N unknowns [9]. The system is represented by the Equation (9).

$$\Psi_i^{n+1} = r\Psi_{i-1}^n + (1-2r)\Psi_i^n + r\Psi_{i+1}^n. \tag{9}$$

2.2. Entropy conservation

Equation (2) is not invariant for the substitution on time t and it describes an irreversible process. The main characteristic of irreversible processes is the approach to a steady state when the time is too long [6]. In the case of the physical phenomenon of heat transfer in a metal bar, physical experience shows that the transient state converges to the steady state [10]. Using the hypothesis that the bar can be divided into $N-1$ parts then the entropy production for each part i is defined by the Equation (10).

$$dE_i = \frac{dS_i}{\Psi_i} \ (i = 1,2, \dots, N-1), \tag{10}$$

where E and S are the entropy and energy of each part.

The entropy variation $\frac{dE}{dt}$ is the sum of two parts, the internal entropy variation and the external entropy variation, the Equation (11) summarizes this behavior (6).

$$\frac{dE}{dt} = \frac{d_i E}{dt} + \frac{d_e E}{dt}. \tag{11}$$

In order to define a mathematical expression to calculate the amount of entropy in each region, it is necessary to calculate a mathematical expression involving the complete Equation (11) together with the temperature function Ψ. For this reason, the Equation (12) and Equation (13) allows to represent the variation of entropy inside and outside in terms of the temperature Ψ function and the rate of change $\frac{Ak}{\Delta x}$ of heat flow in each of its parts [11].

$$\frac{d_i E}{dt} = -\frac{Ak}{\Delta x}\sum_{i=0}^{N-1}(\Psi_{i+1}^n - \Psi_i^n)\left(\frac{1}{\Psi_{i+1}^n} - \frac{1}{\Psi_i^n}\right), \tag{12}$$

$$\frac{d_e E}{dt} = -\frac{Ak}{\Delta x}\frac{1}{\Psi_0}(\Psi_1 - \Psi_0) + \frac{Ak}{\Delta x}\frac{1}{\Psi_N}(\Psi_N - \Psi_{N-1}). \tag{13}$$

Relating the heat flow and Fourier's law leads to the continuity Equation (14) in each part of the metal bar.

$$\frac{dS_i}{dt} = -k\nabla\Psi_i^n. \tag{14}$$

Comparing the complete Equation (2) and Equation (14) generates the Equation (15) relating entropy to temperature function [11].

$$\frac{d\Psi_i^n}{dt} = \frac{\lambda}{(\Delta x)^2}[(\Psi_{i+1}^n - \Psi_i^n) - (\Psi_i^n - \Psi_{i-1}^n)]. \tag{15}$$

3. Results and discussion

Suppose a 10 cm metal bar is placed in a heat source where the temperature at the ends is 320 K and 200 K respectively. The initial temperature inside is 200 K, $A = 1.1\ cm^2$, specific heat $k = 1.1$ cal, $\rho = 10.8\frac{g}{cm^3}$, and $c = 0.03\frac{cal}{g*K}$. By designing a computer program to solve the system of Equation (9), it is possible to calculate the temperature of the bar by simultaneously varying positions and time.

Figure 1 shows the behavior of the temperature profile; from the behavior of Figure 1 it is possible to conclude that the bar temperature tends to stabilize around a linear function. Experimental data in combination with theoretical heat transfer models [12] show that in steady state, heat conduction is a linear function. Therefore, the results generated in Figure 1 are as one would expect. Figure 1 aims to establish that heat conduction energy change in the metal bar tends to stabilize.

Likewise, by complementing the computer program, that generated Figure 1, with the Equation (12), Equation (13), and Equation (15), it is possible to establish the energetic behavior of the physical phenomenon of heat conduction in the metal bar. Figure 2 shows the entropy balance (yellow curve) generated under the equation by adding the internal entropy variation (blue curve) in addition to the external entropy variation (red curve).

From the observation of the entropy balance function (yellow curve) it is possible to observe that the function tends to zero, which guarantees that the system tends to a stable state. In addition, from the external and internal entropy functions represented in Figure 2, by the red and blue curves, it is possible to conclude that the equality $\frac{d_i E}{dt} = -\frac{d_e E}{dt}$ is true. Therefore, by applying the theorem of minimum entropy production [11] the system tends to the steady state.

The analysis of the experimental data taken at the Ladrillera Ocaña, Colombia, [13] allowed to demonstrate by mathematical arguments the behavior of the calculated temperature in the bricks is shown in Figure 1. Therefore, the thermodynamic analysis of the research can be extrapolated to the case of heat transfer at the Ladrillera Ocaña, Colombia.

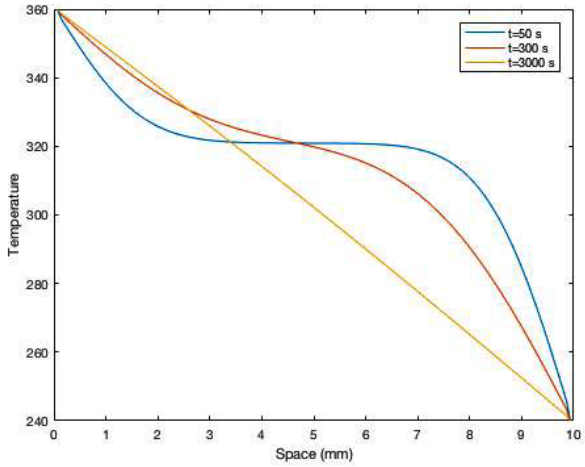


Figure 1. Contour of temperature with the numerical method.

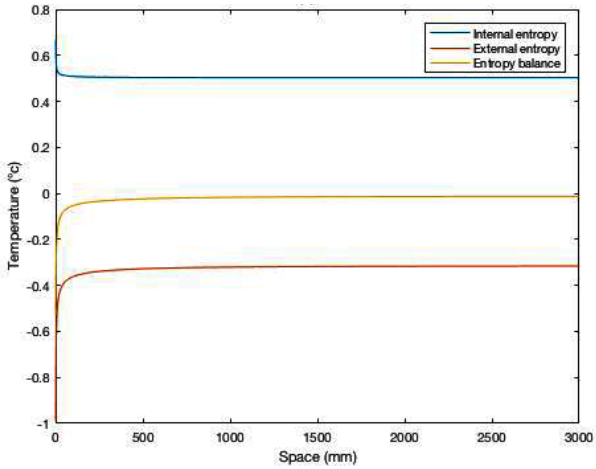


Figure 2. Entropy balance with the numerical method.

4. Conclusion

The energetic behavior of the heat conduction phenomenon in a metal bar was studied in two stages. In the first one, by means of a numerical method that calculated the temperature of the metal bar varying position and time. The numerical results showed that as time increases the temperature tends to a linear function in harmony with experimental and theoretical results. In the second part through the study of the energy balance equation, by means of numerical methods, it was concluded that the system tends to a steady state by using the theorem of minimum entropy production.

References

[1] Van Ness H C 1993 *Understanding Thermodynamics* (New York: Dover Publications)
[2] Cengel Y A, Boles M A, Kanoglu M 2018 *Thermodynamics: An Engineering Approach* (New York: McGraw-Hill Education)
[3] Chen M, Rosendahl L, Bach I, Condra T, Pedersen J 2007 Irreversible transfer processes of thermoelectric generators *American Journal of Physics* **75(9)** 815
[4] Morales A 2009 The second law of classical thermodynamics stated in terms of twin systems *American Journal of Physics* **77(4)** 365
[5] Zurcher U 2014 Thermodynamics of bread baking: A two-state model *American Journal of Physics* **82(3)** 224
[6] Prigogine I 1967 *Introduction to Thermodynamics of Irreversible Processes* (New York: Interscience Publishers)

[7] Logan D 2015 *Applied Partial Differential Equations* (Cham: Springer science)
[8] Smith G D 1985 *Numerical Solution of Partial Differential Equations: Finite Difference Methods* (Oxford: Oxford University Press)
[9] Özişik M N 1994 *Finite Difference Methods in Heat Transfer* (Boca Raton: CRC Press)
[10] Carslow H S, Jaeger J C 1959 *Conduction of Heat in Solids* (Oxford: Clarendon Press)
[11] Lurie D Wagensberg J 1980 Concepts of nonequilibrium thermodynamics in discrete model of heat conduction *American Journal of Physics* **48(10)** 868
[12] Arpaci V 1966 *Conduction Heat Transfer* (Reading: Addison-Wesley Publishing Company)
[13] Nolasco C, Guerrero Gómez G, Gómez J A 2019 Mathematical model of firing process of Ladrillera Ocaña, Colombia *Journal of Physics: Conference Series* **1408(1)** 012017:1